3.2.4.5 *Fatigue life prediction with stable alternating stresses*

Safety factor approach is often used for assessing the fatigue strength of the part in machinery design. Its criterion is described as $S \geqslant [S]$. The first procedure is determining the maximum and minimum stresses σ_{max} and σ_{min} in the part, then calculating the mean stress σ_m and stress amplitude σ_a. Thus the working point $C(\sigma_m, \sigma_a)$ can be shown in the allowable fatigue design diagram. Let $S_{\sigma a}$ and S_σ be the assessed safety factors and $[S]$ be the allowable safety factor. The strength design equations for three common cases (i. e. r, σ_m and σ_{min} are constants respectively) under uniaxial stress situation are briefly given in this section.

(1) In the case of $r=c$ (a constant), the allowable fatigue design diagram is shown in Fig. 3. 6.

When the working points are on fatigue safety zone

$$S_{\sigma a} = \frac{k_N \sigma_{-1}}{(k_\sigma)_D \sigma_a + \psi_\sigma \sigma_m} \geqslant [S_{\sigma a}] \tag{3.34}$$

where ψ_σ is a constant with regarding to the material and can be determined by fatigue tests or following equation

$$\psi_\sigma = \frac{2\sigma_{-1} - \sigma_0}{\sigma_0} \tag{3.35}$$

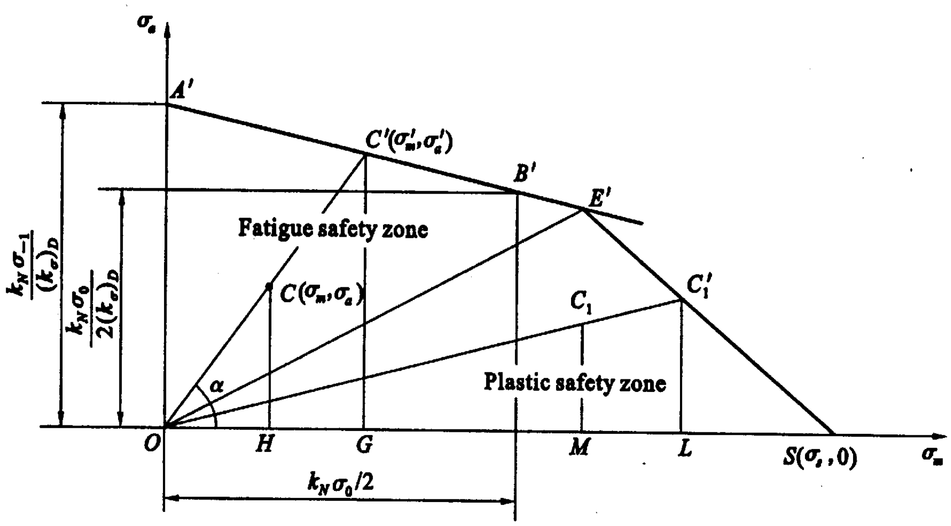


Fig. 3. 6 Allowable fatigue design diagram for case $r=c$

When the working points are on plastic safety zone, the safety factor condition for yielding strength would be

$$S_\sigma = \frac{\sigma_s}{\sigma_a + \sigma_s} \geqslant [S_\sigma] \tag{3.36}$$

(2) In the case of $\sigma_m = c$, the allowable fatigue design diagram is shown in Fig. 3. 7.

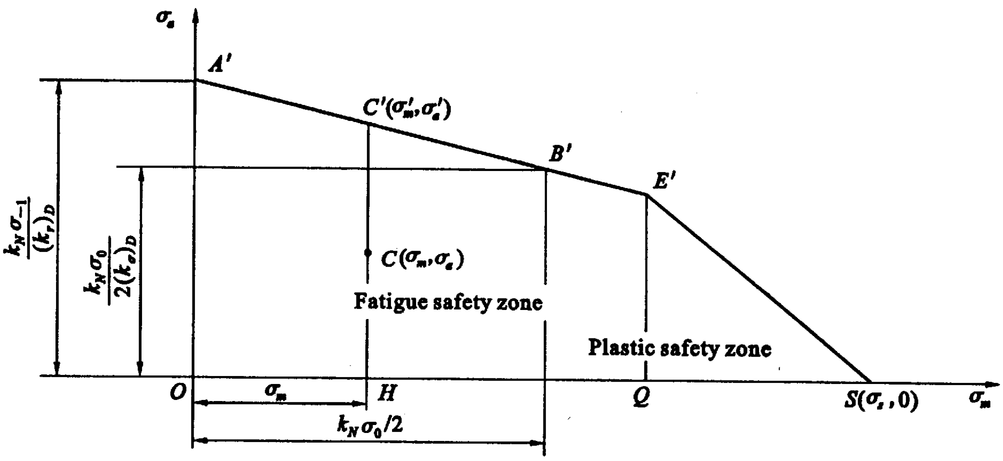


Fig. 3.7 Allowable fatigue design diagram for $\sigma_m = c$

When the working points are on fatigue safety zone

$$S_{\sigma a} = \frac{k_N\sigma_{-1} - \psi_\sigma\sigma_m}{(k_\sigma)_D\sigma_a} \geqslant [S_{\sigma a}] \tag{3.37}$$

When the working points are on plastic safety zone, the safety factor condition for yielding strength would be

$$S_\sigma = \frac{\sigma_s}{\sigma_a + \sigma_m} \geqslant [S_\sigma] \tag{3.38}$$

(3) In the case $\sigma_{\min} = c$, the allowable fatigue design diagram is shown in Fig. 3.8.

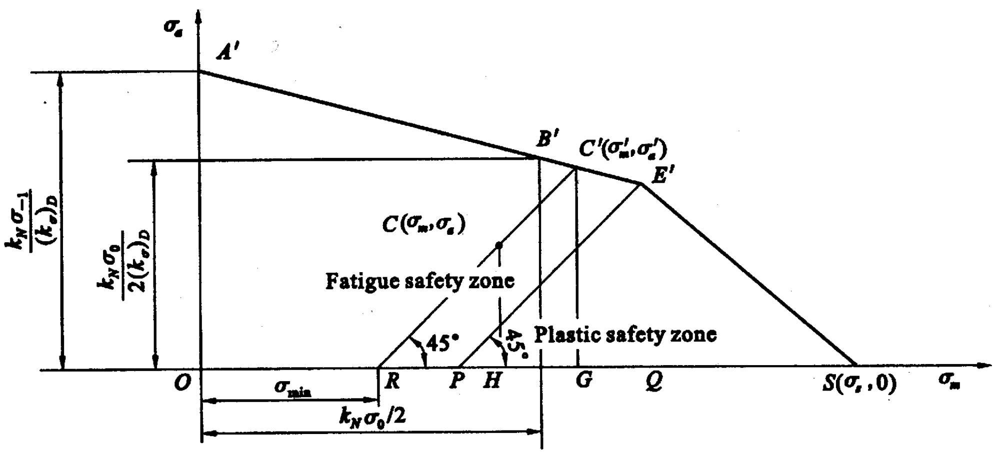


Fig. 3.8 Allowable fatigue design diagram for $\sigma_{\min} = c$

When the working points are on fatigue safety zone

$$S_{\sigma a} = \frac{k_N\sigma_{-1} - \psi_\sigma\sigma_{\min}}{[(k_\sigma)_D + \psi_\sigma]\sigma_a} \geqslant [S_{\sigma a}] \tag{3.39}$$

When the working points are on plastic safety zone, the safety factor condition for yielding strength would be

$$S_\sigma = \frac{\sigma_s}{2\sigma_a + \sigma_{\min}} \geqslant [S_\sigma] \tag{3.40}$$

3.2.4.6 *Prediction of fatigue life with regularly varying stresses*

For the large percentage of mechanical and structural components subjected to randomly varying stress cycle intensity (e. g. , automotive suspension[12] and aircraft structural parts), the prediction of fatigue life is very complicated and has to be done by statistical fatigue strength method based on numerous tests. However, for the components subjected to regular unstable varying stress cycle intensity (e. g. , spindle of machine tools), the prediction of fatigue life can be made by using the *linear cumulative*[13]*-damage rule*, also known as Palmgren's rule or Miner's rule which assumes a linear relation between the degree of damage and the number of cycle to failure. This rule logically proposes a simple concept that if a part is cyclically loaded at a stress level σ_1 causing failure in the number of cycle N_1, assuming that every cycle of this loading σ_1 does the same damage to the part, the damage ratio of each cycle is $1/N_1$. Thus after n_1 cycles, stress level σ_1 does damage n_1/N_1. Similarly, stress level σ_2 at n_2 cycles would do damage n_2/N_2, and so on. When the total damage reaches 100%, fatigue failure occurs.

The linear cumulative-damage rule is expressed by the following equation in which n_1, n_2, $\cdots n_k$ represent the number of cycles at specific stress levels $\sigma_1, \sigma_2, \cdots \sigma_k$, and $N_1, N_2, \cdots N_k$ represent the fatigue lives (in cycles) at these stress levels, as shown in Fig. 3. 9. The stress levels with a value σ_a less than the fatigue limit σ_r are ignored based on the assumption that these stresses will not cause fatigue damage at infinite number of cycles. Fatigue failure is predicted by the linear cumulative-damage rule when

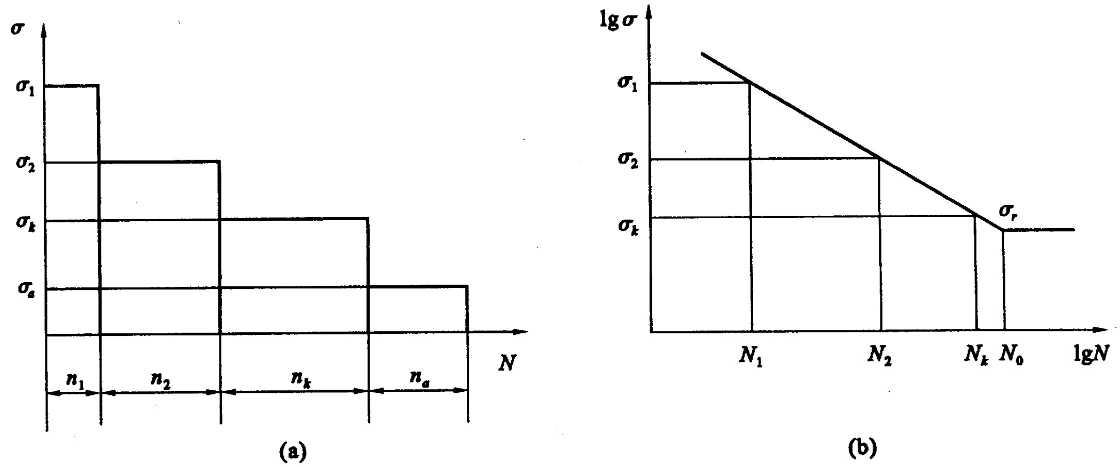


Fig. 3. 9 Fatigue cumulative-damage diagram

(a) Regularly varying stresses; (b) σ-N diagram of the varying stresses

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots + \frac{n_k}{N_k} = 1 \quad \text{or} \quad \sum_{j=1}^{k} \frac{n_j}{N_j} = 1 \tag{3.41}$$

It should be pointed out that although this rule is a useful means to reasonably predict fatigue life, it is only an approximate approach. According to numerous testing data, the rule has been verified only when the stress levels $\sigma_1, \sigma_2, \cdots \sigma_k$ are similar and there is no sudden over-loading. Otherwise, the total damage would be less than 1 when the stress levels loaded are arranged from high to low, larger than 1 when the stress levels loaded are arranged from low to high. It is usually found in engineering that $0.7 \leqslant F \leqslant 2.2$.

3.3 SURFACE STRENGTHS OF THE MACHINE COMPONENTS

Except the damages occurring within the body of a part (fracture, yielding, excessive deflection), various kinds of damage can occur on the surface of a part, which make it unfit for use. Actually, experience indicates that some machinery components, such as a pair of mating gears, ball and roller bearings, friction clutches etc, which function by mating with one another by rolling or/and sliding contact, could fail potentially through surface deterioration than by breakage. The endurance on surface damages is called *surface strengths* of a component, which can be classified as *surface contact strength*, *surface extrusion strength* and *surface wear strength* based on their contact types and working conditions.

3.3.1 Surface Contact Stresses and Strengths

When two elastic bodies with curved surfaces are pressed together, theoretical point or line contact changes to area contact as aresult of the elastic deformations in the contact bodies.

The stresses at the mating surface of curved bodies in compression are called *contact stress* or *Hertz stresses* to honour Heinrich Hertz of Germany because of his original analysis of elastic contact stresses. Here only the two special cases of contacting cylinders and contacting spheres are considered.

When two cylinders of length b and radius ρ_1 and ρ_2 are pressed together with a force F which is uniformly distributed along cylinder length b, the area of contact is a narrow rectangle of width $2a$ and length b. Similarly, when the two contacting bodies are two solid spheres of radius ρ_2 and ρ_1 subjected to a concentrated force F, a circular area of radius c is obtained. In both cases the contact pressure (contact stress) has a semielliptical distribution, i. e. the maximum contact stress $\sigma_{H\max}$ occurs at the centre of the contact area along the force axis as shown in Table 3. 1.

By specifying E_1, μ_1 and E_2, μ_2 as the respective elastic moduli and the Poisson's ratios of the materials for the two contacting bodies, the equations for contact stresses are given in Table 3. 1. It is noticed that the relationship between the maximum contact $\sigma_{H\max}$ and force F is non-linear as there are $\sigma_{H\max} \propto F^{1/2}$ for cylinders and $\sigma_{H\max} \propto F^{1/3}$ for spheres.

The equations can be applied to internal cylinder or sphere contacting by making radius of internal surface as a negative $\rho_2 = -|\rho_2|$. The equations can also be applied to a cylinder or a sphere contacting a plane surface by making $\rho_2 = \infty$ for the plane.

Surface failures at static loads are normally surface crushing for brittle materials and surface plastic deformation for ductile materials. The strength design criterion for this cases is

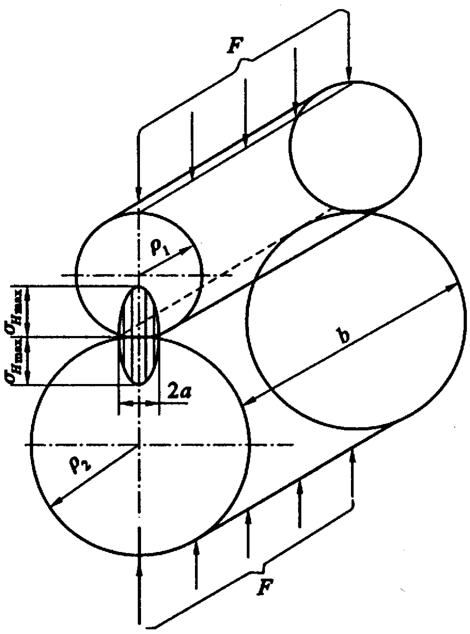
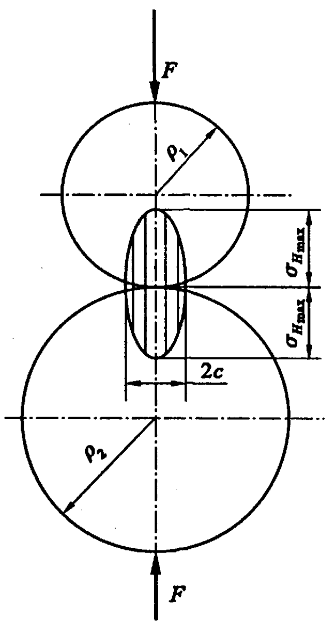
$$\sigma_{H\max} \leqslant [\sigma_H]_{\max} \tag{3.42}$$

where $[\sigma_H]_{\max}$ is the maximum allowable contact stress.

Surface failures at alternating loads normally result from the repeated application of loads that produce stresses on and below the contacting surfaces. A maximum shear stress which has developed about 15～25μm below the contacting surface causes repeated shearing plasticity and initiates fatigue cracks that propagate rapidly to the surface until small bits of surface material become detached, producing *pitting*[14] or *spalling*[15]. For components such as mating gears and ball bearings, pitting and spalling are the common surface fatigue failure. The surface fatigue damages decrease effective working area and the loading capability of the parts and deteriorate the working condition by causing vibration and noise.

Although the exact mechanism of the pitting is not fully understood, it is recognized that the Hertz stresses, the number of cycles, the surface finish, the hardness, the degree of lubrication, and the temperature all influence the strength. The determination of the surface fatigue strength of mating materials is similar to the determination of the fatigue strength of bulk materials as shown in Fig. 3. 3 and Eqs. (3. 28) and (3. 29). Only the fatigue strengths corresponding to the critical number of cycles N are called the *contact endurance limit* or the *contact fatigue limit*, which are set as σ_{HN}.

Table 3. 1 Stress analysis and calculation for contact of two cylinders and two spheres

Two cylinders held in contact by force F (uniformly distributed along cylinder length b)	Two spheres held in contact by force F
	
The half-width a $a=\sqrt{\frac{4F}{\pi b}\left[\frac{\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}}{\frac{1}{\rho}}\right]}$	The contact radius c $c=\sqrt[3]{\frac{3F}{4}\left[\frac{\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}}{\frac{1}{\rho}}\right]}$

The maximum contact stress $\sigma_{H\max}$ $$\sigma_{H\max}=\frac{4}{\pi}\frac{F}{2ab}=\sqrt{\frac{F}{\pi b}\left[\frac{\frac{1}{\rho}}{\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}}\right]}$$ When $\mu_1=\mu_2=0.3$ and $E_1=E_2=E$ $$\sigma_{H\max}=0.418\sqrt{\frac{FE}{b\rho}}$$	The maximum contact stress $\sigma_{H\max}$ $$\sigma_{H\max}=\frac{3}{2}\frac{F}{2\pi c^2}=\frac{1}{\pi}\sqrt[3]{6F\left[\frac{\frac{1}{\rho}}{\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}}\right]^2}$$ When $\mu_1=\mu_2=0.3$ and $E_1=E_2=E$ $$\sigma_{H\max}=0.388\sqrt[3]{\frac{FE^2}{\rho^2}}$$

where ρ is called combined radius of curvature, and $\frac{1}{\rho}=\frac{1}{\rho_1}+\frac{1}{\rho_2}$;

where E is called *combined elastic modulus*, and $E=\frac{2E_1E_2}{E_1+E_2}$.

3.3.2 Surface Extrusion Stresses and Strengths

The stresses occur on the contacting surface when two components contact each other for transmitting loads called *extrusion stresses*. Fig. 3.10 shows a shaft pin connection under axial load F. It is obvious that extrusion stresses arise on the contact surface between the pin and the hole in the shaft, if the stresses exceed the limit of the material.

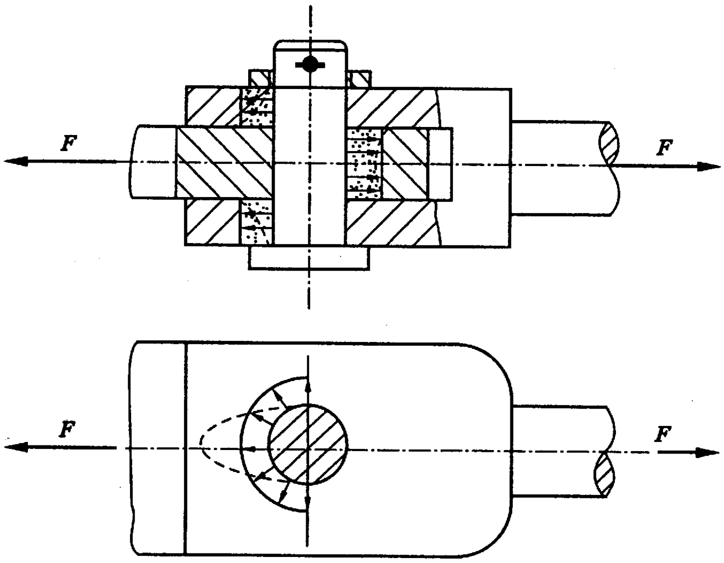


Fig. 3.10 Shaft pin connection

In addition to satisfying other strength requirements, components withstanding extrusion loading must be designed so that extrusion stresses are within acceptable limits. Since the distribution of extrusion stresses is complicated (as marked by dashed line in Fig. 3.10), in engineering, uniformed distributing extrusion stress is assumed (as marked by solid line in Fig. 3.10). A simplified design criterion of extrusion strength is given as

$$\sigma_p=\frac{F}{A}\leqslant[\sigma_p] \tag{3.43}$$

where σ_p and $[\sigma_p]$ are extrusion stress and allowable extrusion stress respectively. A is the area of the contact surface or projection area for the curved contact surface.

3.3.3 Surface Wear Strengths

Wear is a predominant cause of failure of mechanical parts, which work under sliding or rolling contact conditions. Wear may be defined as the removal of material from solid surface as the result of mechanical action. The mechanism of wear is complex which involves many factors such as type of the material, hardness and roughness of the surface, contact pressure, sliding or rolling velocity and all sorts of environmental conditions (moisture, temperature and lubrication, etc.). Thus, the design criteria for wear life are case dependent.

In the case of low sliding or rolling velocity with high contact pressure p, the criterion is

$$p \leqslant [p] \tag{3.44}$$

In the case of mid-high velocity of sliding or rolling, friction energy has to be considered to avoid lubrication failure resulting from high temperature. Since the friction energy is proportional to μpv, when the friction coefficient μ is considered as a constant, the criterion is

$$pv \leqslant [pv] \tag{3.45}$$

while in the case of high speed of sliding or rolling, the velocity has to be a consideration to avoid failure by its effect on accelerating wear process

$$v \leqslant [v] \tag{3.46}$$

where $[p]$, $[pv]$ and $[v]$ are design allowable values and will be given in the design manual.