# Calculating conjugate cam profiles by vector equations

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# Calculating conjugate cam profiles by vector equations

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**Abstract:** This paper presents an analytical approach for determining the profiles of conjugate disc cams. For a conjugate cam mechanism, its two normal lines through the points of contact and the line of centres must always intersect at a common point, which is an instant centre. On this basis, the contact points between the conjugate cam and the follower can be determined according to the locations of instant centres and follower position. The cam profiles, the paths of the cutter and the pressure angles can then be expressed in the form of parametric vector equations. For various types of conjugate cams, the equations for such expressions are formulated, and examples are provided to illustrate the approach. The procedure is especially simple to program.

**Keywords:** conjugate cam profile, instant centre, vector

## NOTATION

| | |
|---|---|
| A | contact point |
| B | contact point |
| C | roller centre |
| $d$ | distance between the roller centres, breadth of the flat-faced follower |
| D | roller centre |
| $e$ | follower offset |
| E | point |
| $f$ | distance from the cam centre to the follower pivot point |
| G | cutter centre |
| H | cutter centre |
| $I_{12}, I_{13}, I_{23}$ | instant centres |
| $\boldsymbol{i}$ | unit vector |
| $\boldsymbol{j}$ | unit vector |
| $l$ | arm length of the follower |
| $L$ | distance of the follower centre from the cam centre measured parallel to the roller translation $= L(\theta)$ |
| $O_2$ | fixed pivot of the cam |
| $O_3$ | fixed pivot of the oscillating follower |
| $q$ | distance from the cam centre to the instant centre $I_{23}$ |
| Q | location of the instant centre $I_{23}$ |
| $r_b$ | base circle radius |
| $r_c$ | cutter radius |
| $r_f$ | radius of the roller follower |
| $S$ | follower motion program $= S(\theta)$ |
| $t$ | time |
| $V_Q$ | speed of point Q |
| $(X, Y)$ | Cartesian coordinate system fixed on the cam |
| $\alpha_A, \alpha_B$ | angles |
| $\eta$ | subtending angle of the follower arms |
| $\theta$ | cam rotation angle |
| $\xi_A$ | angular displacement function of the follower $= \xi_A(\theta)$ |
| $\phi_A, \phi_B$ | pressure angles |
| $\omega_2$ | angular velocity of the cam |

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## 1 INTRODUCTION

In a cam mechanism, the follower must always be held in contact with the cam throughout the motion cycle, and this is usually accomplished by a positive drive or a return spring. A normal conjugate cam mechanism can eliminate the return-spring force and thus result in lower contact stresses, when compared with the spring-loaded type. This important advantage makes it especially suitable for high-speed applications. To perform safely and reliably its intended function, however, the conjugate cams must be properly designed and accurately manufactured. Therefore, the cam profiles and the paths of the cutter centre should be determined analytically.

Hanson and Churchill [**1**], employing the theory of envelopes, presented an analytical method for computing disc–cam profile coordinates. Although the theory of envelopes is not always taught in the college course of calculus, this method has been widely adopted. On the other hand, Davidson [**2**] suggested another method using instant centres, but his contribution seems to have attracted little attention from kinematicians. As a matter of fact, the analytical method using the instant centre approach can provide a convenient means for determining the disc–cam profile and cutter coordinates. In addition, it is applicable to not only the common spring-loaded type but also the conjugate cam mechanism.

## 2 CONJUGATE CAMS WITH AN OFFSET TRANSLATING ROLLER FOLLOWER

Figure 1 shows a conjugate cam mechanism with an offset translating roller follower. There are two cams A and B, fixed on a common shaft. Two follower rollers C and D, mounted to a common follower, are each pushed in opposite directions by the conjugate cams. Setting up a Cartesian coordinate system $(X, Y)$ fixed on the cam and with its origin at the fixed pivot $O_2$, the cam profile coordinates may be expressed in terms of the cam rotation angle $\theta$, which is measured against the direction of cam rotation from the reference radial to cam centreline parallel to roller translation.

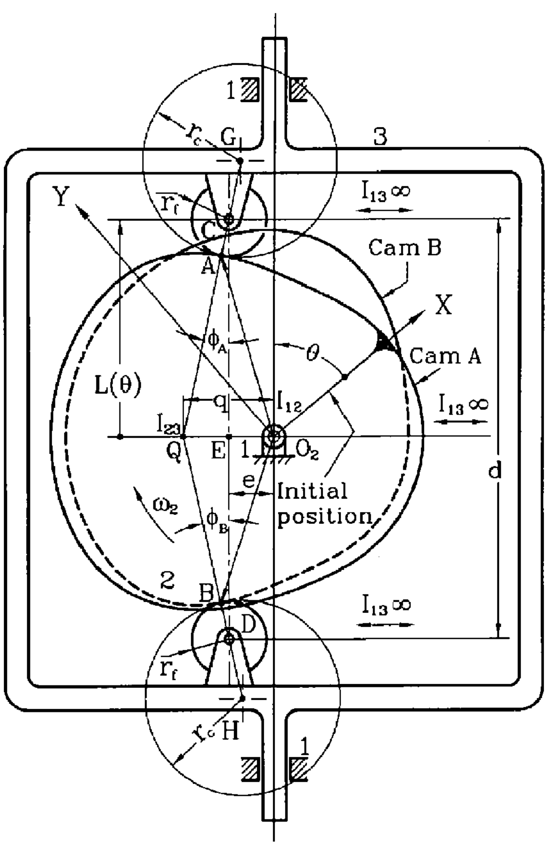


**Fig. 1** Conjugate disc cams with an offset translating roller follower

A conjugate cam mechanism may be considered as a permanent critical form and must always have three velocity instant centres [**3**]. As shown in Fig. 1, this means that the two normal lines through the points of contact and the line of centres must always intersect at a common point, the instant centre $I_{23}$, where I denotes the instant centre and subscripts indicate the related links. For simplicity, in the following, the ground link will be consistently numbered as 1, the cam as 2 and the follower as 3. For the sake of clarity, two other instant centres $I_{12}$ and $I_{13}$ are also located and labelled in the figure. By labelling instant centre $I_{23}$ as Q and $O_2Q = q$, the speed of point Q on the cam can be expressed as

$$V_Q = q\omega_2 \tag{1}$$

where $\omega_2$ is the angular velocity of the cam. In order to let $\theta$ have a counterclockwise angle, in this paper, the cam is to rotate clockwise.

On the other hand, for a translating follower, all points on the follower have the same velocity. Therefore, the speed of point Q on the follower can be expressed as

$$V_Q = \frac{dL(\theta)}{dt} = \frac{dL(\theta)}{d\theta}\frac{d\theta}{dt} = \frac{dL(\theta)}{d\theta}\omega_2 \tag{2}$$

where $L(\theta)$ is the displacement function of the follower:

$$L(\theta) = \sqrt{(r_b + r_f)^2 - e^2} + S(\theta) \tag{3}$$

where $r_b$ is the base circle radius of cam A, $r_f$ is the radius of the roller follower, $e$ is the offset and $S(\theta)$ is the follower motion program. (Since the cam is to rotate clockwise, the quantity $e$ is negative if the offset is to the right; in the position shown it is positive.) By definition of the instant centre, instant centre $I_{23}$ (point Q) is a point common to links 2 (conjugate cam) and 3 (follower) having the same velocity. Therefore, from equations (1) and (2),

$$q = \frac{dL(\theta)}{d\theta} = \frac{dS(\theta)}{d\theta} \tag{4}$$

As a result, after $r_b$, $r_f$, $e$ and $S(\theta)$ have been selected, for each specified value of $\theta$, the roller centre C may be located by application of equation (3) and point Q by application of equation (4).

The pressure angle is the angle between the common normal at the contact point and the direction of motion of the follower [**4**]. For cam A, it is the angle between lines CQ and CE. From △CQE, the pressure angle $\phi_A$ of cam A can be expressed as

$$\phi_A = \tan^{-1}\frac{q - e}{L(\theta)} = \tan^{-1}\left\{\frac{1}{L(\theta)}\left[\frac{dS(\theta)}{d\theta} - e\right]\right\} \tag{5}$$

Therefore, the parametric equations for the profile coordinates of cam A are

$$\boldsymbol{O_2A} = \boldsymbol{O_2E} + \boldsymbol{EC} + \boldsymbol{CA} \tag{6}$$

where

$$\boldsymbol{O_2E} = e\cos(\theta + 90^\circ)\boldsymbol{i} + e\sin(\theta + 90^\circ)\boldsymbol{j} \tag{7}$$

$$\boldsymbol{EC} = L(\theta)\cos\theta\boldsymbol{i} + L(\theta)\sin\theta\boldsymbol{j} \tag{8}$$

$$\boldsymbol{CA} = r_\mathrm{f}\cos(\theta + 180^\circ - \phi_\mathrm{A})\boldsymbol{i} + r_\mathrm{f}\sin(\theta + 180^\circ - \phi_\mathrm{A})\boldsymbol{j} \tag{9}$$

In the same manner, after the distance between roller centres $d$ has been selected, the other roller centre D may be located. From $\triangle$DQE, the pressure angle $\phi_\mathrm{B}$ of cam B can be expressed as

$$\phi_\mathrm{B} = \tan^{-1}\frac{q-e}{d-L(\theta)} = \tan^{-1}\left\{\frac{1}{d-L(\theta)}\left[\frac{\mathrm{d}S(\theta)}{\mathrm{d}\theta} - e\right]\right\} \tag{10}$$

Similarly, the parametric equations for the profile coordinates of cam B are

$$\boldsymbol{O_2B} = \boldsymbol{O_2E} + \boldsymbol{ED} + \boldsymbol{DB} \tag{11}$$

where

$$\boldsymbol{ED} = [d - L(\theta)]\cos(\theta + 180^\circ)\boldsymbol{i} + [d - L(\theta)]\sin(\theta + 180^\circ)\boldsymbol{j} \tag{12}$$

$$\boldsymbol{DB} = r_\mathrm{f}\cos(\theta + \phi_\mathrm{B})\boldsymbol{i} + r_\mathrm{f}\sin(\theta + \phi_\mathrm{B})\boldsymbol{j} \tag{13}$$

In practice, the cutter or grinding wheel is frequently chosen larger than the follower roller for reasonable grinding wear life [**5**]. The locations of the cutter centres for cutting cams A and B are also shown in Fig. 1. Because the cutter and roller centres must lie on a common normal to the cam profile [**5**], normally outward extending the cam profile by a length of cutter radius $r_\mathrm{c}$ obtains the location of the cutter centre. In other words, for cam A, the cutter centre G and points Q, A and C must always lie on a line. Therefore, the parametric equations for the coordinates of the cutter centre G are

$$\boldsymbol{O_2G} = \boldsymbol{O_2E} + \boldsymbol{EC} + \boldsymbol{CG} \tag{14}$$

where

$$\boldsymbol{CG} = (r_\mathrm{c} - r_\mathrm{f})\cos(\theta - \phi_\mathrm{A})\boldsymbol{i} + (r_\mathrm{c} - r_\mathrm{f})\sin(\theta - \phi_\mathrm{A})\boldsymbol{j} \tag{15}$$

The location of the cutter centre H for cutting cam B can also be located in the same way:

$$\boldsymbol{O_2H} = \boldsymbol{O_2E} + \boldsymbol{ED} + \boldsymbol{DH} \tag{16}$$

where

$$\boldsymbol{DH} = (r_\mathrm{c} - r_\mathrm{f})\cos(\theta + 180^\circ + \phi_\mathrm{B})\boldsymbol{i} + (r_\mathrm{c} - r_\mathrm{f})\sin(\theta + 180^\circ + \phi_\mathrm{B})\boldsymbol{j} \tag{17}$$

In fact, the conjugate cam profiles of Fig. 1 have been drawn, by applying these equations, to meet the following requirements. The follower is to rise 20 mm with cycloidal motion while the cam rotates clockwise from 0° to 100°, dwell for the next 50°, return with cycloidal motion for 100° cam rotation and dwell for the remaining 110°. Both follower rollers have the same radius of 10 mm. The offset $e$ is 12 mm and the distance between the roller centres, $d$, is 113 mm. The base circle radius of cam A is 40 mm.

## 3 CONJUGATE CAMS WITH A TRANSLATING FLAT-FACED FOLLOWER

Figure 2 shows a conjugate cam mechanism with a translating flat-faced follower. Setting up a Cartesian coordinate system ($X$, $Y$) fixed on the cam and with its

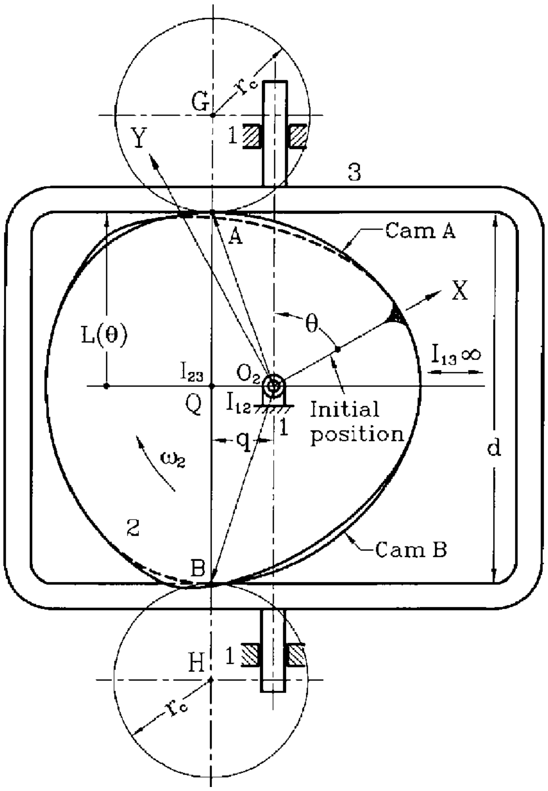


**Fig. 2** Conjugate disc cams with a translating flat-faced follower

origin at the fixed pivot $O_2$, the cam profile coordinates may be expressed in terms of $\theta$.

This mechanism may also be considered as a permanent critical form and must always have three instant centres. It follows that the two normal lines and the line of centres must always intersect at a common point, the instant centre $I_{23}$. In this case, it also means that the two contact points A and B and instant centre $I_{23}$ must always lie on a vertical line. By labelling the instant centre $I_{23}$ as Q and $O_2Q = q$, the speed of point Q on the cam can be expressed as

$$V_Q = q\omega_2 \tag{18}$$

The speed of point Q on the follower can be expressed as

$$V_Q = \frac{dL(\theta)}{dt} = \frac{dL(\theta)}{d\theta}\frac{d\theta}{dt} = \frac{dL(\theta)}{d\theta}\omega_2 \tag{19}$$

where $L(\theta)$ is the displacement function of the follower:

$$L(\theta) = r_b + S(\theta) \tag{20}$$

where $r_b$ is the base circle radius of cam A and $S(\theta)$ is the follower motion program. From equations (18) and (19),

$$q = \frac{dL(\theta)}{d\theta} = \frac{dS(\theta)}{d\theta} \tag{21}$$

As a result, after $r_b$ and $S(\theta)$ have been selected, for each specified value of $\theta$, point Q may be located by means of the corresponding value of $q$, which can be found from equation (21), and then the contact point A may be located by means of the corresponding value of $L(\theta)$. Therefore, the parametric equations for the profile coordinates of cam A are

$$\boldsymbol{O_2A} = \boldsymbol{O_2Q} + \boldsymbol{QA} \tag{22}$$

where

$$\boldsymbol{O_2Q} = q\cos(\theta + 90°)\boldsymbol{i} + q\sin(\theta + 90°)\boldsymbol{j} \tag{23}$$

$$\boldsymbol{QA} = L(\theta)\cos\theta\boldsymbol{i} + L(\theta)\sin\theta\boldsymbol{j} \tag{24}$$

Similarly, the parametric equations for the profile coordinates of cam B are

$$\boldsymbol{O_2B} = \boldsymbol{O_2Q} + \boldsymbol{QB} \tag{25}$$

where

$$\boldsymbol{QB} = [d - L(\theta)]\cos(\theta + 180°)\boldsymbol{i} + [d - L(\theta)]\sin(\theta + 180°)\boldsymbol{j} \tag{26}$$

and $d$ is the breadth of the follower.

As previously indicated, normally outward extending the cam profile by a length of cutter radius $r_c$ obtains the location of the cutter centre. Therefore, for cam A, the parametric equations for the coordinates of the cutter centre G are

$$\boldsymbol{O_2G} = \boldsymbol{O_2Q} + \boldsymbol{QG} \tag{27}$$

where

$$\boldsymbol{QG} = [L(\theta) + r_c]\cos\theta\boldsymbol{i} + [L(\theta) + r_c]\sin\theta\boldsymbol{j} \tag{28}$$

The location of the cutter centre H for cutting cam B can also be located in the same way:

$$\boldsymbol{O_2H} = \boldsymbol{O_2Q} + \boldsymbol{QH} \tag{29}$$

where

$$\boldsymbol{QH} = [d - L(\theta) + r_c]\cos(\theta + 180°)\boldsymbol{i} + [d - L(\theta) + r_c]\sin(\theta + 180°)\boldsymbol{j} \tag{30}$$

The conjugate cam profiles shown in Fig. 2 have been drawn, by applying these equations, to meet the following requirements. The follower is to rise 22 mm with cycloidal motion while the cam rotates clockwise from 0° to 140°, dwell for the next 60°, return with cycloidal motion for 100° cam rotation and dwell for the remaining 60°. The base circle radius of cam A, $r_b$, is 40 mm and the breadth of the follower $d$ is 102 mm.

## 4 CONJUGATE CAMS WITH AN OSCILLATING ROLLER FOLLOWER

Figure 3 shows a conjugate cam mechanism with an oscillating roller follower. In this case, $f$ represents the distance from the cam centre to the follower pivot point and $l$ represents the arm length of the follower. Setting up a Cartesian coordinate system $(X, Y)$ fixed on the cam and with its origin at the fixed pivot $O_2$, the cam profile coordinates may be expressed in terms of $\theta$,

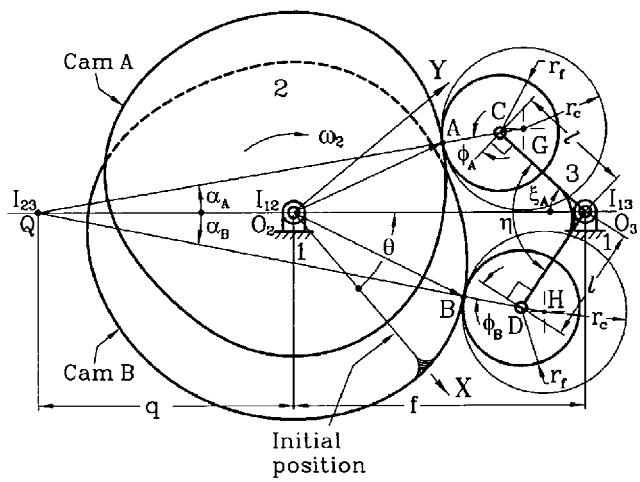


**Fig. 3** Conjugate disc cams with an oscillating roller follower

which is measured against the direction of cam rotation from the reference radial on the cam to the line between the cam centre and follower pivot point.

Because it is a permanent critical form, its two normal lines and the line of centres must always intersect at the instant centre $I_{23}$, point Q. The speed of point Q on the cam can be expressed as

$$V_Q = q\omega_2 \tag{31}$$

where $q = O_2Q$ and $\omega_2$ is the angular velocity of the cam. On the other hand, the speed of point Q on the follower can be expressed as

$$V_Q = (f+q)\frac{d\xi_A(\theta)}{dt} = (f+q)\frac{d\xi_A(\theta)}{d\theta}\omega_2 \tag{32}$$

where $\xi_A(\theta)$ is the angular displacement function of the follower A:

$$\xi_A(\theta) = \cos^{-1}\left[\frac{l^2 + f^2 - (r_b + r_f)^2}{2lf}\right] + S(\theta) \tag{33}$$

where $r_b$ is the base circle radius of cam A, $r_f$ is the radius of the roller follower and $S(\theta)$ is the follower angular motion program. From equations (31) and (32) and after some algebraic manipulation,

$$q = \frac{f\dfrac{d\xi_A(\theta)}{d\theta}}{1-\dfrac{d\xi_A(\theta)}{d\theta}} = \frac{f\dfrac{dS(\theta)}{d\theta}}{1-\dfrac{dS(\theta)}{d\theta}} \tag{34}$$

From $\triangle O_3QC$ and the cosine law,

$$QC = \sqrt{l^2 + (f+q)^2 - 2l(f+q)\cos\xi_A(\theta)} \tag{35}$$

From $\triangle O_3QD$ and the cosine law,

$$QD = \sqrt{l^2 + (f+q)^2 - 2l(f+q)\cos[\eta - \xi_A(\theta)]} \tag{36}$$

where $\eta$ is the subtending angle of the follower arms. From $\triangle O_3QC$ and the sine law,

$$\alpha_A = \sin^{-1}\left[\frac{l\sin\xi_A(\theta)}{QC}\right] \tag{37}$$

From $\triangle O_3QD$ and the sine law,

$$\alpha_B = \sin^{-1}\left\{\frac{l\sin[\eta - \xi_A(\theta)]}{QD}\right\} \tag{38}$$

Therefore, the parametric equations for the profile coordinates of cam A are

$$\boldsymbol{O_2A} = \boldsymbol{O_2Q} + \boldsymbol{QA} \tag{39}$$

where

$$\boldsymbol{O_2Q} = q\cos(\theta + 180^\circ)\boldsymbol{i} + q\sin(\theta + 180^\circ)\boldsymbol{j} \tag{40}$$

$$\boldsymbol{QA} = (QC - r_f)\cos(\theta + \alpha_A)\boldsymbol{i} + (QC - r_f)\sin(\theta + \alpha_A)\boldsymbol{j} \tag{41}$$

The parametric equations for the profile coordinates of cam B are

$$\boldsymbol{O_2B} = \boldsymbol{O_2Q} + \boldsymbol{QB} \tag{42}$$

where

$$\boldsymbol{QB} = (QD - r_f)\cos(\theta - \alpha_B)\boldsymbol{i} + (QD - r_f)\sin(\theta - \alpha_B)\boldsymbol{j} \tag{43}$$

From $\triangle O_3QC$, the pressure angle $\phi_A$ of cam A can be expressed as

$$\phi_A = 90^\circ - \alpha_A - \xi_A(\theta) \tag{44}$$

From $\triangle O_3QD$, the pressure angle $\phi_B$ of cam $B$ can be expressed as

$$\phi_B = 90^\circ - \alpha_B - [\eta - \xi_A(\theta)] \tag{45}$$

As previously indicated, the cutter and roller centre must lie on a common normal to the cam profile. Therefore, normally outward extending the cam profile by a length of cutter radius $r_c$ obtains the location of the cutter centre. In other words, for cam A, the cutter centre G and points Q, A and C must always lie on a line. As a result, the parametric equations for the coordinates of the cutter centre G are

$$\boldsymbol{O_2G} = \boldsymbol{O_2Q} + \boldsymbol{QG} \tag{46}$$

where

$$\boldsymbol{QG} = (QC - r_f + r_c)\cos(\theta + \alpha_A)\boldsymbol{i} + (QC - r_f + r_c)\sin(\theta + \alpha_A)\boldsymbol{j} \tag{47}$$

The location of the cutter centre H for cutting cam B can also be located in the same way:

$$\boldsymbol{O_2H} = \boldsymbol{O_2Q} + \boldsymbol{QH} \tag{48}$$

where

$$\boldsymbol{QH} = (QD - r_f + r_c)\cos(\theta - \alpha_B)\boldsymbol{i} + (QD - r_f + r_c)\sin(\theta - \alpha_B)\boldsymbol{j} \tag{49}$$

The conjugate cam profiles of Fig. 3 have been drawn, by applying these equations, to meet the following requirements. The follower is to oscillate 30° clockwise with cycloidal motion while the cam rotates clockwise from 0° to 120°, dwell for the next 40°, return with cycloidal motion for 120° cam rotation and dwell for the remaining 80°. The distance between pivots, $f$, is 80 mm.

Both follower arms have the same length of 32 mm and both follower rollers have the same radius of 16 mm. The base circle radius of cam A, $r_b$, is 57.32 mm and the subtending angle of the follower arms, $\eta$, is 100°.

## 5 CONJUGATE CAMS WITH AN OSCILLATING FLAT-FACED FOLLOWER

Figure 4 shows a conjugate cam mechanism with an oscillating flat-faced follower. In this case, $f$ represents the distance from the cam centre to the follower pivot point and $e$ represents the follower face offset from the follower pivot point. (The quantity $e$ is positive in Fig. 4. If the follower face is offset from the pivot point towards the cam centre, it is negative.) Setting up a Cartesian coordinate system $(X, Y)$ fixed on the cam and with its origin at the fixed pivot $O_2$, the cam profile coordinates may be expressed in terms of $\theta$.

This is also a permanent critical form; its two normal lines and the line of centres must always intersect at the instant centre Q. The speed of point Q on the cam can be expressed as

$$V_Q = q\omega_2 \tag{50}$$

where $q = O_2Q$. The speed of point Q on the follower

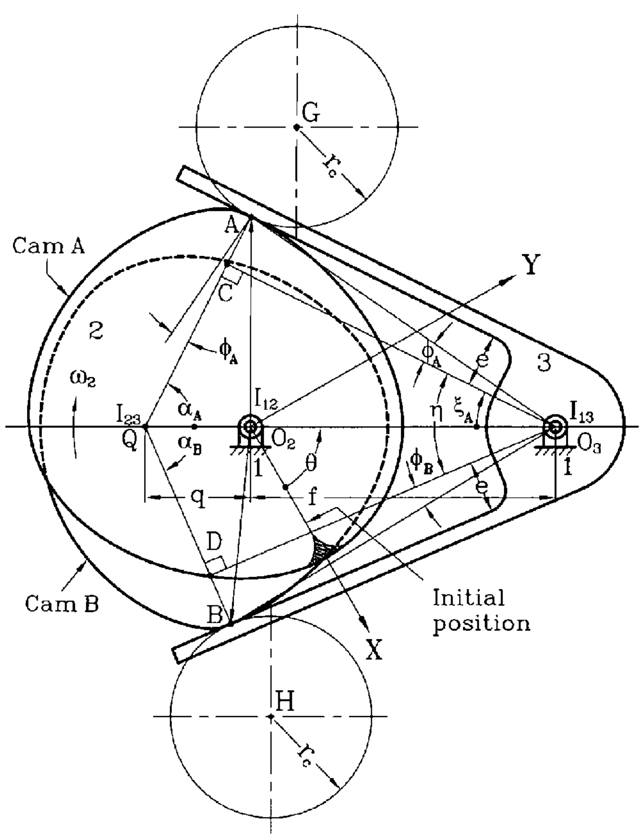


**Fig. 4** Conjugate disc cams with an oscillating flat-faced follower

can be expressed as

$$V_Q = (f + q)\frac{d\xi_A(\theta)}{dt} = (f + q)\frac{d\xi_A(\theta)}{d\theta}\omega_2 \tag{51}$$

where $\xi_A(\theta)$ is the angular displacement function of the follower A:

$$\xi_A(\theta) = \sin^{-1}\left(\frac{r_b - e}{f}\right) + S(\theta) \tag{52}$$

where $r_b$ is the base circle radius of cam A, $e$ is the offset of the follower and $S(\theta)$ is the follower angular motion program. From equations (50) and (51) and after some algebraic manipulation,

$$q = \frac{f\dfrac{d\xi_A(\theta)}{d\theta}}{1 - \dfrac{d\xi_A(\theta)}{d\theta}} = \frac{f\dfrac{dS(\theta)}{d\theta}}{1 - \dfrac{dS(\theta)}{d\theta}} \tag{53}$$

From $\triangle O_3QC$,

$$QC = (f + q)\sin\xi_A(\theta) \tag{54}$$

$$\alpha_A = 90° - \xi_A(\theta) \tag{55}$$

From $\triangle O_3QD$,

$$QD = (f + q)\sin[\eta - \xi_A(\theta)] \tag{56}$$

$$\alpha_B = 90° - [\eta - \xi_A(\theta)] \tag{57}$$

where $\eta$ is the subtending angle of the follower arms. Therefore, the parametric equations for the profile coordinates of cam A are

$$\boldsymbol{O_2A} = \boldsymbol{O_2Q} + \boldsymbol{QA} \tag{58}$$

where

$$\boldsymbol{O_2Q} = q\cos(\theta + 180°)\boldsymbol{i} + q\sin(\theta + 180°)\boldsymbol{j} \tag{59}$$

$$\boldsymbol{QA} = (QC + e)\cos(\theta + \alpha_A)\boldsymbol{i} + (QC + e)\sin(\theta + \alpha_A)\boldsymbol{j} \tag{60}$$

The parametric equations for the profile coordinates of cam B are

$$\boldsymbol{O_2B} = \boldsymbol{O_2Q} + \boldsymbol{QB} \tag{61}$$

where

$$\boldsymbol{QB} = (QD + e)\cos(\theta - \alpha_B)\boldsymbol{i} + (QD + e)\sin(\theta - \alpha_B)\boldsymbol{j} \tag{62}$$

From $\triangle O_3AC$, the pressure angle $\phi_A$ of cam A can be expressed as

$$\phi_{\mathrm{A}} = \tan^{-1}\left[\frac{e}{(f+q)\cos\xi_{\mathrm{A}}(\theta)}\right] \tag{63}$$

From $\triangle \mathrm{O_3BD}$, the pressure angle $\phi_{\mathrm{B}}$ of cam B can be expressed as

$$\phi_{\mathrm{B}} = \tan^{-1}\left\{\frac{e}{(f+q)\cos[\eta-\xi_{\mathrm{A}}(\theta)]}\right\} \tag{64}$$

As previously indicated, normally outward extending the cam profile by a length of cutter radius $r_{\mathrm{c}}$ obtains the location of the cutter centre. Therefore, the parametric equations for the coordinates of the cutter centre G for cutting cam A are

$$\boldsymbol{O_2G} = \boldsymbol{O_2Q} + \boldsymbol{QG} \tag{65}$$

where

$$\begin{aligned}\boldsymbol{QG} = {} & (\mathrm{QC} + e + r_{\mathrm{c}})\cos(\theta + \alpha_{\mathrm{A}})\boldsymbol{i} \\ & + (\mathrm{QC} + e + r_{\mathrm{c}})\sin(\theta + \alpha_{\mathrm{A}})\boldsymbol{j}\end{aligned} \tag{66}$$

The location of the cutter centre H for cutting cam B can also be located in the same way:

$$\boldsymbol{O_2H} = \boldsymbol{O_2Q} + \boldsymbol{QH} \tag{67}$$

where

$$\begin{aligned}\boldsymbol{QH} = {} & (\mathrm{QD} + e + r_{\mathrm{c}})\cos(\theta - \alpha_{\mathrm{B}})\boldsymbol{i} \\ & + (\mathrm{QD} + e + r_{\mathrm{c}})\sin(\theta - \alpha_{\mathrm{B}})\boldsymbol{j}\end{aligned} \tag{68}$$

The conjugate cam profiles of Fig. 4 have been drawn, by applying these equations, to meet the following requirements. The follower is to oscillate 15° clockwise with cycloidal motion while the cam rotates clockwise from 0° to 120°, dwell for the next 50°, return with cycloidal motion for 100° cam rotation and dwell for the remaining 90°. The distance between pivots, $f$, is 80 mm. Both follower arms have the same offset of 14 mm. The base circle radius of cam A, $r_{\mathrm{b}}$, is 40 mm and the subtending angle of the follower arms, $\eta$, is 50°.

## 6 DISCUSSION AND CONCLUSION

Another topic that is frequently encountered in the cam design is the determination of cam curvature. The designed cam profile may have distortion under certain conditions. However, after the parametric equations describing the cam profile have been developed, the curvature of the cam profile can be calculated accurately [**5–7**]. When undercutting occurs, the radius of curvature switches sign from positive to negative. As a consequence, the potential distortion of the cam profile may be checked analytically.

A conjugate cam mechanism may be considered as a permanent critical form, and the two normal lines through the points of contact and the line of centres must always intersect at the instant centre $\mathrm{I_{23}}$. For a cam mechanism with specified system parameters, follower dimensions and cam–follower motion program, the three instant centres of the mechanism can be located. According to the locations of instant centres and follower position, the contact points between the cam and the follower, the pressure angles and the locations of the cutter centre can be determined and expressed in the form of parametric vector equations.

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## REFERENCES

1 **Hanson, R. S.** and **Churchill, F. T.** Theory of envelopes provides new cam design equations. *Product Engng*, 20 August 1962, 45–55.
2 **Davidson, J. K.** Calculating cam profiles quickly. *Mach. Des.*, 7 December 1978, 151–155.
3 **Paul, B.** *Kinematics and Dynamics of Planar Machinery*, 1979, pp. 285–289 (Prentice-Hall, Englewood Cliffs, New Jersey).
4 **Martin, G. H.** *Kinematics and Dynamics of Machines*, 2nd edition, 1982, p. 212 (McGraw-Hill, New York).
5 **Rothbart, H. A.** *Cams: Design, Dynamics, and Accuracy*, 1956, pp. 78–89 (John Wiley, New York).
6 **Molian, S.** *The Design of Cam Mechanisms and Linkages*, 1968, pp. 64–70 (American Elsevier, New York).
7 **Jensen, P. W.** *Cam Design and Manufacture*, 2nd edition, 1987, pp. 94–105 (Marcel Dekker, New York).