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Tracking Control and Reference Trajectory Generation in a LOM System

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An improvement in contour accuracy is acquired by the introduction of a tracking controller and a trajectory generation policy in a laminated object manufacture (LOM) system. A model of the positioning system is developed as the design basis of the tracking controller. A zero phase error tracking controller (ZPETC) is used to eliminate single axis following error, and thus reduce the contour error. Then the cross-coupled controller (CCC) is introduced to further minimise the contour error caused by the two-axis mismatch; the method of off-line generating of two-dimension reference trajectories followed by the positioning system is proposed. Simulation is developed using a Matlab model and satisfactory results are obtained.

Keywords: Contour error; LOM; Tracking control; Trajectory generation

1. Introduction

Laminated object manufacture (LOM) technology is widely used in industry. It makes possible that a manufacturer can develop a model or prototype directly from a CAD design in a short time, which significantly reduces the design time and time to market. Therefore, LOM technology has been primarily used for design conceptualisation, assembly verification and simulation [1]. As the application of LOM technology expands from merely prototyping to functional parts, the fabrication of accurate geometry in a shorter time becomes an important issue. In the case of LOM applications, it depends on the reference trajectory generation and tracking control as well as on the acceleration capability of the machine. In addition, The contour errors generated by the build process are essentially due to the *x–y* positioning errors which involve the contribution of servo mechanism and the track control algorithm. The focus of this paper will be particularly oriented to tracking control and reference trajectory generation in a LOM system. To evaluate the performance of the proposed approach, simulation is developed using a Matlab model based on a retrofitted LOM machine in the Kinergy Mech Co. Ltd of Singapore.

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2. LOM Technology

2.1 LOM Process Description

The LOM process is a laminated manufacturing technique for the fabrication of paper parts. The process starts by designing a boundary surface model of the part using computer-aided design (CAD) software. The CAD model is then converted into a stereolithographic (STL) file which approximates the surfaces of the model by triangular polygons. The STL model is then sliced into a succession of horizontal layers ordered by their coordinates on the *z*-axis. For each layer the boundaries of the slices are defined by contours (i.e. tool-paths). The contour information is then downloaded to the machine.

2.2 Machine Description

The heart of a LOM machine is the *x–y* gantry positioner, which is specially designed for high-speed and high-precision laser cutting. Figure 1 shows an *x–y* gantry positioner in the ZIPPYI system developed by Kinergy Mech of Singapore. A laser head is fixed on the slider of axis *y*. The *x–y* gantry

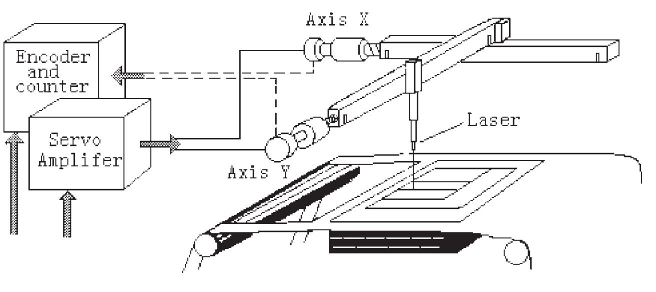


Fig. 1. *x–y* gantry positioner in LOM machine.

positioner drives the laser head along the contour (tool-path) on the paper. The movements are executed according to the contour generated from the slicing process. Once a layer is built, the platform moves down one paper thickness in the z-direction and another paper is laid on by a roller to form the following layer and then in sequence, up to the top layer of the part.

3. Model of the Positioning System and Its Controller Structure

3.1 Model of the DC Servomotor

In our studied framework, the positioning system is a linear open-frame x–y table that uses high-precision ball screws and DC servomotors. A suitable preload is applied to the ball screw to maintain high stiffness and no backlash. Two incremental encoders are directly coupled to the servomotors and are used for the velocity and position feedback. The dynamic Eq. of the DC servomotor can be reduced to Eq. (1) [2] according to the schematic diagram shown in Fig. 2:

$$V_a = R_a I_a + L_a \frac{dI_a}{dt} + K_b\omega \qquad (1)$$

$$T = T_L + B\omega + J_e \frac{d\omega}{dt}$$

$$T = K_T I_a$$

where V_a(V) = armature voltage
I_a(A) = armature current
$R_a(\Omega)$ = winding resistance
L_a(mH) = winding inductance
K_b(V/(rad/s)) = voltage constant
K_T(N·m/A) = torque constant
J_e(Kg·m^2) = equivalent rotor moment of inertia
ω(rad/s) = motor rotational speed
B(N·m/(rad/s)) = damping
T_L(N·m) = load torque (including friction torque)

In generally, the winding inductance and the friction torque can be ignored. Therefore taking the Lapace transform of (1), the transform function of the DC servomotor is given as

$$\frac{\theta(s)}{V(s)} = \frac{K_m}{s(T_m s + 1)} \qquad (2)$$

or

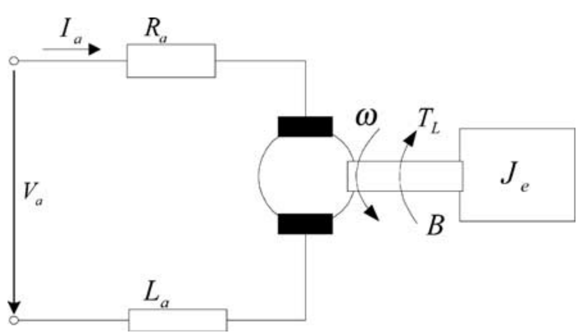


Fig. 2. Schematic diagram of the DC servomotor.

$$\frac{\omega(s)}{V(s)} = \frac{K_m}{T_m s + 1}$$

Where

$$K_m = \frac{K_T}{(BR_a + K_T K_b)},\ T_m = \frac{J_e R_a}{(BR_a + K_T K_b)}$$

3.2 Controller Structure

Figure 3 shows the basic schematic of the proposed x–y positioner tracking control structure in the LOM system. A tracking feedforward compensator is added in order to compensate for the dynamic response and impose an appropriate tracking input between the new reference (desired) input $X_r(k)$ and the actual output $X(k)$. Adding a feedforward compensator in the axis controller can improve the response of the single axis, and consequently compensate for the path following error of each individual axis, but it cannot reduce the resultant contour error due to (1) mismatch in axial-loop parameters, (2) external disturbances and (3) the contour shape in nonlinear trajectories and corners [3,4], so another coordination compensator is introduced.

As shown in Fig. 1, the load on the x-axis is larger than that on the y-axis. Moreover, in LOM applications, the laser motion trajectory is mostly nonlinear and involves many sharp corners. All these factors have harmful effects on the contour control. To further minimize the contour error owing to the mismatch of the 2-axis parameters and the contour shape in nonlinear trajectories and corners, and as we know the contour error is more important than the path error, a cross-coupled controller (CCC) is used. The CCC was first proposed by Koren [5]. The main idea is to build in real time a contour error model based on the feedback information from the two axes as well as from the trajectories generator, to find an optimal compensating law, and then to feedback correction signals to the individual axes. The cross-coupling controller includes two major parts:

1. The real-time contour error model.
2. A control law.

In general, real-time contour error compensation is cost-effective but difficult. In this paper, a real-time contour error compensation technology referred to in [6] is applied to estimate the magnitude of the contour error and determine the compensation components for each axes.

Figure 4 shows the detail structure of the x-axis controller (the y-axis controller structure is similar).

In Fig. 4, A_x represents the amplifier gain, $K_{vx}$is the analogue velocity feedrate gain, and K_{px} is the discrete position feedback gain. The discrete transfer function between the input $u_x(k)$ of the velocity loop and the position output $x(k)$ is given by

$$G_x(z) = \frac{x(z)}{u(z)} = \frac{b_0 z - b_1}{(z-1)(z-p)} \qquad (3)$$

where

$$b_0 = \frac{K_x}{a}\left(T + \frac{1}{a}(p-1)\right),$$

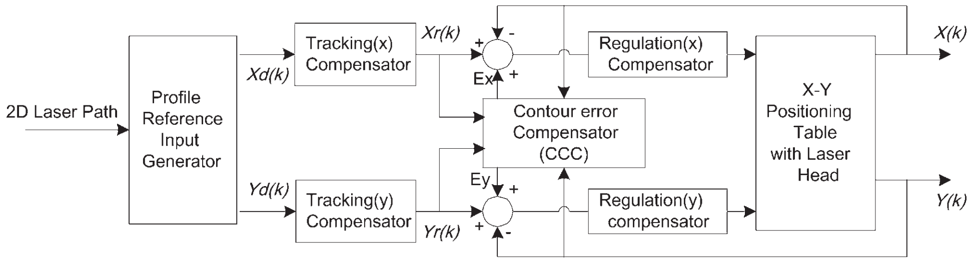


Fig. 3. Proposed *x–y* positioner tracking control structure.

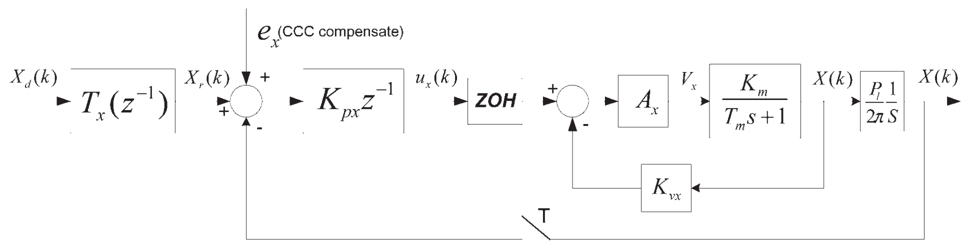


Fig. 4. *X*-axis controller structure.

$$b_1 = \frac{K_x}{a}\left(pT + \frac{1}{a}(p-1)\right)$$

$$p = exp(-aT) ,$$

$$a = \frac{1}{T_m}(A_x K_T K_{vx} + 1) ,$$

$$K_x = \frac{1}{2\pi T_m} A_x K_m P_l$$

The proportional and derivative gains were chosen for near critically damped closed-loop behaviour. Without considering the CCC compensation, the closed-loop transfer function between the desired input $x_r(k)$ and the actual position output $x(k)$ is given by

$$G_r(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = K_{px} \frac{z^{-2}(b_0 - b_1 z^{-1})}{1-(1+p)z^{-1}+(p+K_{px}b_o)z^{-2}-K_{px}b_1 z^{-3}} \tag{4}$$

$G_r(z^{-1})$ has two complex conjugate poles and one negative real pole close to the origin of the z-plane. The feedforward compensator $T_x(z^{-1})$ compensates the closed-loop transfer function $G_r(z^{-1})$ and imposes a faster tracking signal. However, because the zero of $G_r(z^{-1})$ ($z_1 = -0.82$) is negative, it should not be cancelled, otherwise, it will result in a lightly damped oscillatory output. To ensure good tracking performance without cancelling this closed-loop zero, a zero phase error tracking controller (ZPETC) was proposed by Tomizuka [7] in the context of welding applications. In the ZPETC [8], the compensator $T_x(z^{-1})$ cancels the regulation dynamics defined by $A(z^{-1})$ and any stable zeros in $B(z^{-1})$. It also applies a feedforward dynamic scale factor $\frac{B(z)}{[B(1)]^2}$ to guarantee zero phase errors between the reference input position $x_d(k)$ and the actual position $x(k)$. In this case, the tracking compensator will be given by

$$T_x(z^{-1}) = \frac{A(z^{-1})B(z)}{[B(1)]^2} = \frac{(b_0 z^{-1} - b_1)(1-(1+p)z^{-1}+(p+K_{px}b_0)z^{-2}-K_{px}b_1 z^{-3})}{K_{px}(b_0 - b_1)^2 z^{-2}} \tag{5}$$

The compensation is realisable, because the desired input sequence $x_d(k)$ and, in particular, the two-step look-ahead value, are available well in advance. Hence, at instant kT, the feedforward control action will depend on the two-step look-ahead value of $x_d(k)$, shown by

$$x_r(k) = \frac{(b_0 z^{-1} - b_1)(1-(1+p)z^{-1}+(p+K_{px}b_0)z^{-2}-K_{px}b_1 z^{-3})}{K_{px}(b_0 - b_1)^2} x_d(k+2) \tag{6}$$

The transfer function (7) between the reference trajectory input $x_d(k)$ and the plant output $x(k)$ without CCC compensation indicates that the plant output is a moving average of the desired trajectory with a unit steady-state gain.

$$G_f(z) = \frac{(b_0 z^{-1} - b_1)(b_0 - b_1 z^{-1})}{(b_0 - b_1) \quad (b_0 - b_1)} \tag{7}$$

The transfer function between the plant output $x(k)$ and e_x from CCC is also $G_r(z^{-1})$. When the CCC compensation is considered, the plant output is given by

$$x(k) = \frac{(b_0 z^{-1} - b_1)(b_0 - b_1 z^{-1})}{(b_0 - b_1) \quad (b_0 - b_1)} x_d(k) + K_{px} \frac{z^{-2}(b_0 - b_1 z^{-1})}{1-(1+p)z^{-1}+(p+K_{px}b_0)z^{-2}-K_{px}b_1 z^{-3}} e_x(k) \tag{8}$$

4. Reference Trajectory Generation for Tracking Control

Unlike computer numerical control (CNC) technology, which according to the geometrical information provided to it, uses linear, circular, or spline interpolators, in LOM technology only linear interpolation is used (since only line segments are generated after STL stereolithographic slicing). Linear interpolation contributes to the low feedrate, tracking errors and contour errors, which are responsible for the precision of the final part. Therefore, a given layer, including the contours, can be looked upon as a set of line segments. To keep the laser power constant along each line segment, it is necessary to maintain a constant ratio between the digital/analogue (*D/A*) output of the laser and the laser feedrate during the cut process.

$$A_{\text{laser}} = \frac{A_{max}}{V_{max}} \cdot V_{\text{laser}} \tag{9}$$

where V_{laser} (mm/s) is the laser feedrate, i.e. the positioning speed, A_{laser} (V) is the laser D/A output, $A_{\max}$ and $V_{\max}$ are the maximum laser D/A output and laser feedrate respectively. With the current state of technology, the laser output can only be controlled in an open loop without any feedback, and the D/A output delay varied. It is, therefore, essential to keep the feedrate of the laser constant. This constitutes a constraint for the trajectory generating policy [9].

Another constraint on the trajectory generation is the fact that the laser feedrate is not continuous. At the connecting point between two segments, an infinite acceleration or torque is sometimes needed to exactly follow the reference geometry (at sharp corners). However, since the torque is finite, the linear system model shown as Eq. (1) can represent a "good" approximation of the real system only when there is no saturation,

$$\left| J_x \frac{d\omega_x}{dt} + B\omega_x \right| \le T_{\max} \tag{10}$$

At sharp corners, contour errors cannot be avoided unless the laser head stops at the turning points. However, it is not good to stop and start at each corner, especially when the segments are very small, because this will produce a very slow average speed, and also because of the difficulty of controlling the laser output at the stop/start points. Instead of a complete stop, it is more appropriate to reduce the speed within a certain tolerance with respect to Eq. (9).

The trajectory generation is, in fact, an optimisation problem of speed and the required machining time for no jerk in the motion, subject to the two constraints discussed above. Unlike many applications, real time and online trajectory planning is essential. In LOM application, trajectory generation and tracking control can be separated. Therefore an offline trajectory generation model is proposed.

From the two constraints described above, one could conclude that the laser feedrate must be as constant as possible, except at the sharp corners. Indeed, if the laser feedrate were sufficiently low, one would then prescribe constant speed. However, for higher reference speed requirements and sharp corners or very small line segments, constant speed may not be appropriate. To "optimise" the speed curve within a given line segment, it is necessary to take into account in advance of the length of the next segment, and the sharpness of the next corner, so that enough time is given for acceleration or deceleration. This is the main trust of the look-ahead policy that we propose for LOM applications.

The look-ahead algorithm determines the starting and the ending speeds for the current segment, according to the lengths of the current and the next segment, and according to the angle between these two segments. For example, if the angle is close to 180º, the two segments can be regarded as one line segment and no deceleration is needed at the turn point. If the angle is between two predefined values is, for example, between 100º and 176º (this value should be fine-tuned according to simulation result), the maximal permitted feedrate at the turn point should be prescribed. Otherwise, the laser feedrate is reduced according to the sharpness of the corner, especially the sharp corners. The above algorithm uses only linear interpolators. The flowchart in Fig. 5 describes the computational steps required in this algorithm. Different acceleration/deceleration profiles can be chosen (such as trapezoid, S-curve or polynomial function) at each line segment. In this application, a trapezoid profile has been used in order

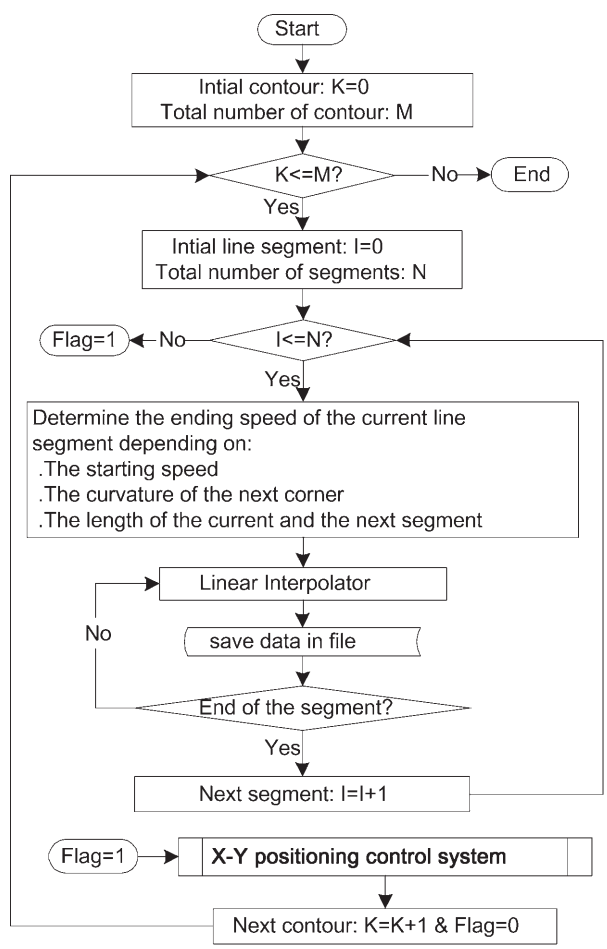


Fig. 5. Flowchart of the proposed trajectory generation algorithm.

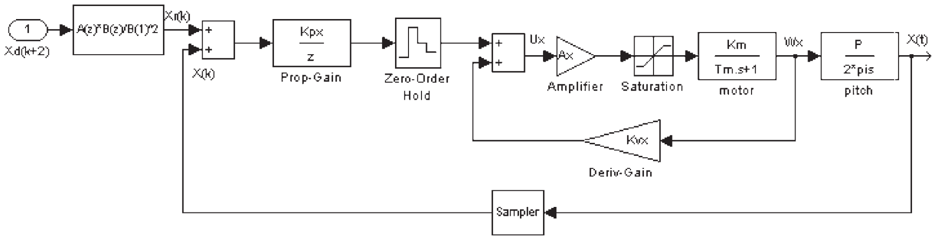


Fig. 6. Matlab simulation model of *x*-axis.

Table 1. Parameters of the *x*-axis drive system.

Parameter	Value
B	4×10^{-4} *N·m/(rad/s)*
R_a	3.2 Ω
K_b	5.5×10^{-2} *V/(rad/s)*
L_a	2 *mH*
J_e	10.4×10^{-5} *Kg·m*^{2}
K_T	6×10^{-5} *N·m/A*
P_l	20 *mm*
A_x	0.005 *V/count*
K_{vx}	0.0117 *V/(ras/s)*
K_{px}	2.083 *(rad/s)/count*

to reduce the complexity of the calculation and the large quantities of memory required.

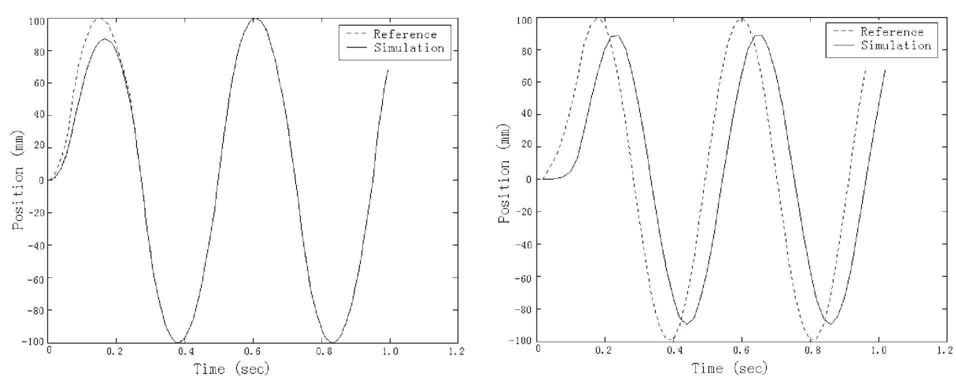
The results of the trajectory generation are saved in a virtual disc as points (x, y) sequences, and then they are fed to the x–y positioning control system described in Section 3 for tracking control.

5. Simulation Result

A simulation model of the two-axis positioning system described above was developed in Matlab in order to evaluate the performance of the reference trajectory generation algorithms in Section 4 and the tracking controller presented in Section 3. Figure 6 shows the Matlab simulation model for the *x*-axis, and the parameters of *x*-axis are shown in Table 1. The model of the positioning system, the regulation feedback and the feedforward tracking compensator is described in Section 3. The simulation model takes into account torque and controller saturation.

To evaluate the performance of the feedforward tracking controller, a "high"-frequency sinusoidal reference input position was applied to the simulation model, with and without the presence of the tracking compensator. Figure 7(*a*) and (*b*) represents the reference input position (dashed line) and the system output position (solid line) obtained, respectively, with and without the tracking compensator. These results show the improvement, in terms of tracking performance, produced by the introduction of the feedforward compensator.
As expected, with this compensator the tracking response of the closed-loop system is faster and can follow the high-speed reference input without introducing any lag.

Figure 8 shows the simulation model for the x–y positioning system, and the individual axis drive system is involved in this model as a subsystem. The CCC compensator plays an important role in this system. The simulation sample shown in Fig. 9 is a circle-like curve that consists of 181 line segments which are interpolated using the proposed trajectory generating method. The result is saved in a file "laserpath.mat" as the reference input, then it is fed to the positioning system. In Fig. 9(*a*) and (*b*), the tracking position (solid line) with compensator and without compensator are compared with the reference position (dotted line) respectively. Figure 9(*c*) shows the contour error without compensator, with ZPETC and with CCC+ZPETC. The result shows that the x–y positioning system



(*a*) (*b*)

Fig. 7. (*a*) With tracking compensator; (*b*) without tracking compensator.

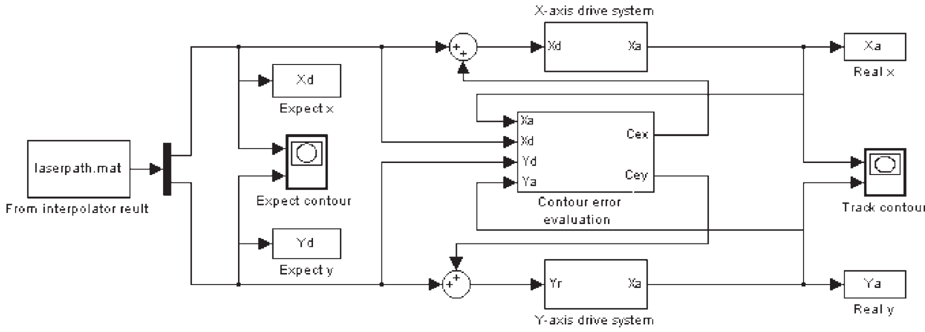


Fig. 8. Simulation model of *x–y* positioning axis.

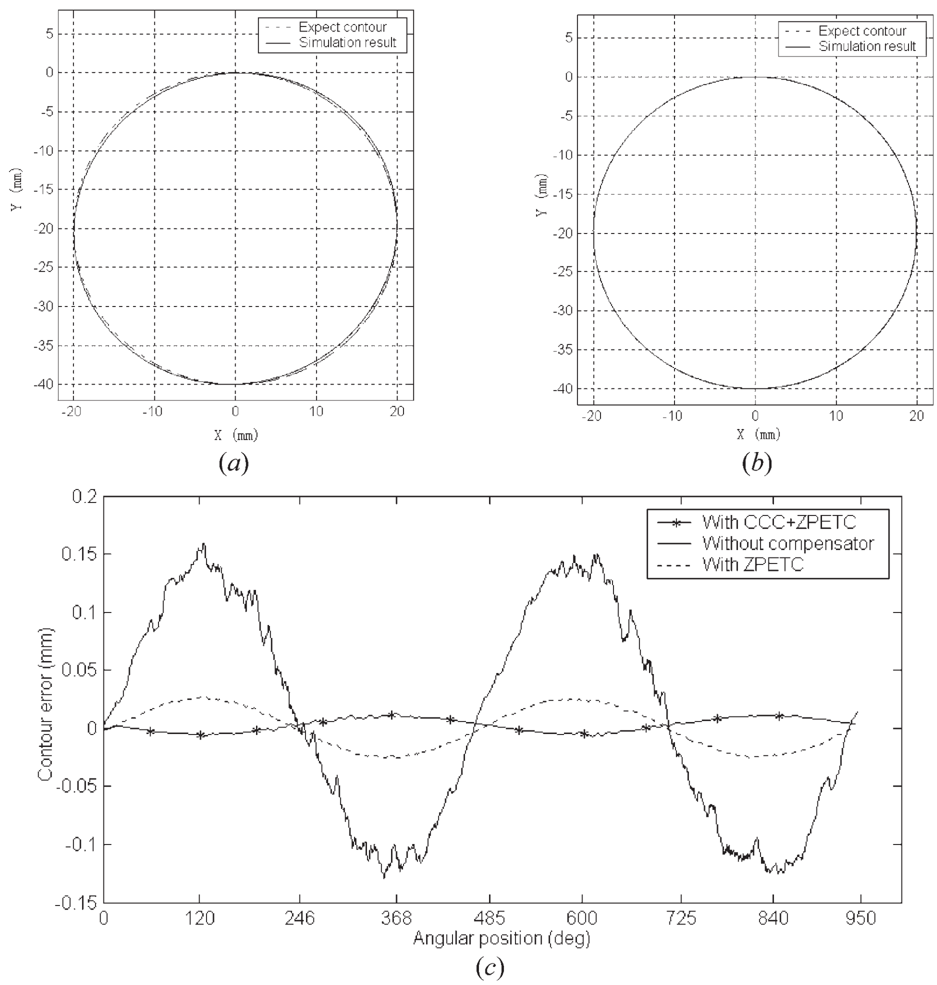


Fig. 9. (*a*) Tracking a circle-like curve withour compensator; (*b*) with tracking compensator. (*c*) Contour error. (Circle r = 20 mm, feedrate 200 mm/s.).

achieves precise contour control with the ZPETC compensator, which reduces the contour error by eliminating the single axis lag, while the CCC+ZPETC can further minimise the contour error caused by the two-axis mismatch.

Conclusion

The control of a LOM system requires essentially a high-precision positioning system together with an accurate laser output. The proposed trajectory generation method is useful for generating a desired trajectory position, which is followed by the positioning system with a ZPETC and a CCC compensator. The ZPETC compensator is used to eliminate the single axis following error, thus reduce the contour error. The CCC is introduced to further minimise the contour error caused by the two-axis mismatch. According to the simulation of the *x*-axis drive system and the *x*–*y* position system on a Matlab model based on a retrofitted LOM machine in Kinergy Mech Co. Ltd of Singapore, a good performance is produced.

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