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A second-generation wavelet-based finite element method for the solution of partial differential equations

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ABSTRACT

A new second-generation wavelet (SGW)-based finite element method is proposed for solving partial differential equations (PDEs). An important property of SGWs is that they can be custom designed by selecting appropriate lifting coefficients depending on the application. As a typical problem of SGW algorithm, the calculation of the connection coefficients is described, based on the equivalent filters of SGWs. The formulation of SGW-based finite element equations is derived and a multiscale lifting algorithm for the SGW-based finite element method is developed. Numerical examples demonstrate that the proposed method is an accurate and effective tool for the solution of PDEs, especially ones with singularities.

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1. Introduction

In recent years, wavelet methods have been developed as a new powerful tool for mathematical analysis and engineering computation since the multiresolution properties and various basis functions of wavelets can lead to fast, hierarchical and accurate algorithms. The current wavelet-based numerical algorithms include wavelet–Galerkin [1,2], wavelet–collocation [3,4], wavelet–finite element [5,6] type, etc. As a general numerical method, the wavelet-based finite element method adopts scaling and wavelet functions instead of traditional polynomial interpolation. Since traditional wavelets are constructed from scaled and shifted versions of a single mother wavelet on a regularly spaced grid over a theoretically infinite or periodic domain, traditional wavelets cannot be constructed on complicated, irregularly spaced meshes, which are commonly encountered in the finite element method.

The second-generation wavelets (SGWs) based on a lifting scheme [7] were introduced to eliminate the restrictions and deficiencies of traditional wavelets. The lifting scheme provides users with much flexibility for building different SGW bases with prediction and update coefficients for engineering problems depending on the applications. In the recent applications of the SGW method in engineering computations, different kinds of wavelets have been designed for grid interpolation and multiscale computation [8–10]. A typical problem in the numerical analysis of wavelet methods is the calculation of the connection coefficient of SGWs, which is an integral of products of wavelet scaling functions or derivative operators associated with these. However, for SGWs lacking explicit function expressions, traditional numerical integrals such as Gauss integrals cannot provide the desirable precision. In the last few decades, the computation of connection coefficients of wavelets without explicit function expressions has been receiving much attention. The algorithms for computing the connection coefficients on unbounded domains or periodic boundary conditions were developed by Cohen, Dahmen, Beylkin et al. [11–13], in order to solve partial differential equations. To apply the wavelet method to the solution of finite domain

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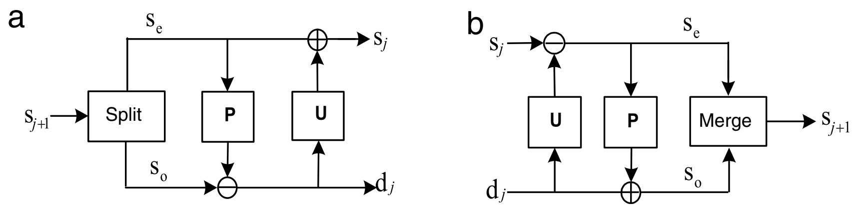

Fig. 1. Second-generation wavelet transform: (a) decomposition; (b) reconstruction.

problems, connection coefficients with integral form on the intervals [0, x] and [0, 2^j] were respectively presented by Monasse and Lin et al. [14,15]. With the development of wavelet methods for general mathematical or engineering problems, connection coefficients with integral form on the interval [0, 1] were proposed by Ko, Chen, Ma et al. [16–18], to improve the efficiency and accuracy of the multiscale computation.

In this work, the relation between lifting coefficients and wavelet filters is described, based on the second-generation wavelet transform (SGWT). The connection coefficients on the interval [0, 1] are calculated efficiently, based on the equivalent filters of SGWs. The SGW-based finite element equations are formulated in order to calculate all kinds of stiffness matrices and load vectors that are necessary for higher-dimensional mathematical problems. A multiscale lifting algorithm for the SGW-based finite element method is also presented. Some numerical examples with high gradients and singularities are presented to verify the accuracy and efficiency of the proposed method.

2. The second-generation wavelet transform

A lifting scheme was presented by Sweldens in order to custom design a new family of SGWs. Fig. 1 shows the decomposition and reconstruction of the SGWT. Consider a signal: $X = \{x_k, k \in Z\}$, $k = 1, 2, \ldots, L$. The approximation signal $\left\{s_{j+1}(k)\right\}$ of $\mathbf{X}$ at scale $j+1$ is split into two disjoint sets, namely even indexed samples $\left\{s_j(2k)\right\}$ and odd indexed samples $\left\{s_j(2k+1)\right\}$. Then, a predictor is used to predict the odd indexed samples $\left\{s_j(2k+1)\right\}$ with N neighbors of the even indexed samples at scale j, and N is determined as the required number of vanishing moments of the underlying wavelet function. The errors in prediction are defined as the detail signal at scale j in the form

$$d_j(k) = s_{j+1}(2k+1) - \sum_{m=1}^{N} p(m) s_{j+1}(2m+k-N), \quad k = 1, 2, \ldots, L/2, \; m = 1, 2, \ldots, N \tag{1}$$

where $p(m)$ is a prediction coefficient and $P = [p(1), \ldots, p(N)]$ denotes the predictor for detail signal calculation. Then a number $\tilde{N}$ of detail signals $d_j(k)$ obtained from Eq. (1) are adopted to update the even indexed samples $\left\{s_j(2k)\right\}$, and the approximation signal $\left\{s_j(k)\right\}$ is

$$s_j(k) = s_{j+1}(2k) - \sum_{n=1}^{\tilde{N}} u(n) d_j(n+k-\tilde{N}/2-1), \quad k = 1, 2, \ldots, L/2, \; n = 1, 2, \ldots, \tilde{N} \tag{2}$$

where $u(n)$ is an update coefficient and $U = \left[u(1), \ldots, u(\tilde{N})\right]^T$, denotes the updater for the approximation signal calculation. The underlying wavelet of the wavelet transform via the lifting scheme can be denoted as $(N, \tilde{N})$. Fig. 2 shows an SGW with the predictor order $N = 4$ and the updater order $\tilde{N} = 4$. Rearranging Eqs. (1) and (2), we find the relation between $d_j(k)$ and $s_{j+1}(k)$, $s_j(k)$ and $s_{j+1}(k)$ as follows:

$$d_j(k) = \sum_{l} g(2k-l) s_{j+1}(l) \tag{3}$$

$$s_j(k) = \sum_{l} h(2k-l) s_{j+1}(l) \tag{4}$$

where $g(2k-l)$ is an equivalent high-pass filter coefficient of the SGWT, the high-pass filter is

$$g = \{g_k, -N+1 \le k \le N-1, k \in Z\} \tag{5}$$

where $g(2k-1) = -p(k)$ and $g(2k) = \delta(k-N/2)$ for $k = 1, 2, \ldots, N$, and $\delta(k)$ is the Dirac function; $h(2k-l)$ is the equivalent low-pass filter coefficient of SGWT, the low-pass filter is

$$h = \left\{h_k, -N-\tilde{N}+2 \le k \le N+\tilde{N}-2\right\} \tag{6}$$

where $h(2k-1) = \delta(k-z) - \sum_{m=1}^{\tilde{N}} p(m)u(k-m+1)$ for $k = 1, 2, \ldots, \tilde{N}$ and $z = (N+\tilde{N})/2$; $h(2k) = u(k)$ for $k = 1, 2, \ldots, \tilde{N}$. On the basis of the flexible design of the prediction and update coefficients, we can calculate the wavelet filters and define SGWs according to the application.

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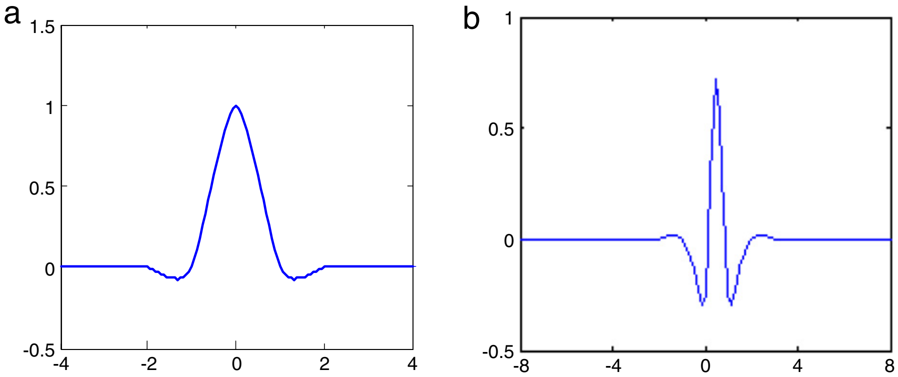

Fig. 2. Second-generation wavelet: (a) scaling functions SGW(4); (b) wavelet functions SGW(4, 4).

3. Connection coefficients

Since the wavelet numerical method can be viewed as a method in which the approximating function is defined by use of a multiresolution technique, the computation of connection coefficients is based on the multiresolution analysis of the wavelets and scaling functions. While using the scaling function of SGW as a test function for the finite element method, we would obtain two typical connection coefficients on the interval [0, 1] for forming stiffness matrices and load vectors [5,18], such as

$$\Lambda_{N,k,l}^{j,a,b} = \int_{-\infty}^{+\infty} \chi_{[0,1]}(\xi)\phi_{j,k}^{(a)}\phi_{j,k}^{(b)}d\xi \tag{7}$$

$$R_{N,k}^{j,c} = \int_{-\infty}^{+\infty} \chi_{[0,1]}(\xi)\xi^{c}\phi_{j,k}d\xi \tag{8}$$

where $\chi_{[0,1]}(\xi) = \begin{cases} 1 & 0 \le \xi \le 1 \\ 0 & \text{otherwise} \end{cases}$, which satisfies a simple two-scale relation

$$\chi_{[0,1]}\left(\frac{1}{2}\xi\right) = \chi_{[0,1]}(\xi) + \chi_{[1,2]}(\xi) = \chi_{[0,1]}(\xi) + \chi_{[0,1]}(\xi - 1). \tag{9}$$

On the basis of the multiresolution analysis of SGWs [7], the scaling function $\phi_{j,k} \subset L^2(R)$ at level j and $j+1$ satisfies the two-scale relation

$$\phi_{j,k} = \sum_{l} \lambda_{j,k,l}\phi_{j+1,l}, \tag{10}$$

where $\lambda_{j,k,l}$ denote the low-pass filters of SGWs relating the filters in the SGWT with $\lambda_j(1-k) = (-1)^k g_j(k)$. By using the two-scale relation of Eqs. (9) and (10), the connection coefficient matrix can be derived as

$$(2^{m+n-1}G - I)\Lambda_N^{j,m,n} = 0 \tag{11}$$

where G is the coefficient matrix, I an identity matrix,

$$G = \sum_{s,t}(\lambda_{s-2k}\lambda_{t-2l} + \lambda_{s-2k+2^j}\lambda_{t-2l+2^j}) \tag{12}$$

where $-(2N-1) \le k, l \le 2^j - 1$, and $\Lambda_N^{j,m,n}$ denotes the $(2^j + 2N - 1) \times (2^j + 2N - 1)$ stiffness matrix. Eq. (7) cannot be determined uniquely through the homogeneous equation (11), so independent inhomogeneous equations are required for a unique solution as follows:

$$\frac{q!}{(q-m)!}\frac{w!}{(w-n)!}\frac{2}{q+w-m-n+1} = 2^{j(m+n)}\sum_{k,l} C_{j,k}^{q} C_{j,l}^{w} \Lambda_{N,k,l}^{j,m,n} \tag{13}$$

where $C_{j,k}^q = \langle x^q, \phi_{j,k}\rangle$. For the computation of the connection coefficients of load vectors, the multiresolution analysis of the SGWs will derive the following equation:

$$(2^{m+1}I - B)R_{N,k}^{j,m} = \sum_{i} \lambda_{i-2k+2^j} \sum_{s=1}^{m} \binom{m}{s} R_{N,i}^{m-s} \tag{14}$$

where $-(2N-1) \le k \le 2^j - 1$, $B = \sum_{i,k}\left(\lambda_{i-2k} + \lambda_{i-2k+2^j}\right)$.

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4. The SGW-based finite element method

Consider a boundary value problem. Suppose that a spatial domain $\Omega \in R^2$ has a Lipschitz boundary $\bar{\boldsymbol{\Gamma}} = \bar{\boldsymbol{\Gamma}}_D \cup \bar{\boldsymbol{\Gamma}}_N$ where $\bar{\boldsymbol{\Gamma}}_D$ and $\bar{\boldsymbol{\Gamma}}_N$ are the Dirichlet and Neumann boundaries, respectively. The boundary value problem consists of finding the solution u that satisfies

$$Lu := -(Au')' + bu' + cu = f \quad \text{in } \Omega \tag{15}$$

$$u = 0 \quad \text{on } \bar{\boldsymbol{\Gamma}}_D \tag{16}$$

$$\sigma u = t \quad \text{on } \bar{\boldsymbol{\Gamma}}_N \tag{17}$$

where L is in principle a second-order differential operator and σ is the boundary operator. The data are assumed to be sufficiently smooth, i.e. A, b, c, f and t are sufficiently regular functions. For a variational formulation of Eq. (13), we introduce the bilinear form as

$$a(u, v) := \int_{\Omega} A(x)u'(x)v'(x) + b(x)u'(x)v(x) + c(x)u(x)v(x)dx \tag{18}$$

$$l(v) := \int_{\Omega} f(x)v dx. \tag{19}$$

If the unknown field function u is interpolated using the SGWs, the field function can be

$$u(\xi) = \boldsymbol{\Phi}_N \boldsymbol{a}^e \tag{20}$$

where ξ is the nondimensional element coordinate with $0 \le \xi \le 1$. $\boldsymbol{\Phi}_N = \left[\phi^j_{N,-2N+1}(\xi), \phi^j_{N,-2N}(\xi), \ldots, \phi^j_{N,2^j-1}(\xi)\right]$ is the row vector combined by the second-generation scaling functions for order N at the scale j, and $\boldsymbol{a}^e = \left[a^j_{N,-2N+1}, a^j_{N,-2N}, \ldots, a^j_{N,2^j-1}\right]^T$ is the column vector of coefficients. Since the elemental nodal solution $\boldsymbol{u}^e$ and scaling function coefficients $\boldsymbol{a}^e$ have the relation

$$\boldsymbol{u}^e = \boldsymbol{R}^e \boldsymbol{a}^e \tag{21}$$

where $\boldsymbol{R}^e = \left[\boldsymbol{\Phi}^T(\xi_1)\boldsymbol{\Phi}^T(\xi_2)\cdots\boldsymbol{\Phi}^T(\xi_{n+1})\right]^T$, the field function can be written as

$$u(\xi) = \boldsymbol{\Phi}_N(\boldsymbol{R}^e)^{-1}\boldsymbol{u}^e = \boldsymbol{\Phi}_N \boldsymbol{T}^e \boldsymbol{u}^e \tag{22}$$

where the transformation matrix $\boldsymbol{T}^e = (\boldsymbol{R}^e)^{-1}$.

Substituting Eqs. (18) and (19) with Eq. (22), we obtain

$$\boldsymbol{K}^e = \int_0^1 \left[A(\xi)(\boldsymbol{T}^e)^T \frac{d\boldsymbol{\Phi}^T}{d\xi}\frac{d\boldsymbol{\Phi}}{d\xi}\boldsymbol{T}^e + b(\xi)(\boldsymbol{T}^e)^T\frac{d\boldsymbol{\Phi}^T}{d\xi}\boldsymbol{\Phi}\boldsymbol{T}^e + c(\xi)(\boldsymbol{T}^e)^T\boldsymbol{\Phi}^T\boldsymbol{\Phi}\boldsymbol{T}^e\right]d\xi \tag{23}$$

$$\boldsymbol{P}^e = l_e \int_0^1 f(\xi)(\boldsymbol{T}^e)^T\boldsymbol{\Phi}^T d\xi. \tag{24}$$

Therefore, solving equations by the SGW-based finite element method can be represented as

$$\boldsymbol{K}^e\boldsymbol{u}^e = \boldsymbol{P}^e \tag{25}$$

where the boundary condition can be treated as in the traditional finite element method.

5. The multiscale lifting algorithm

The multiresolution analysis property of SGWs is the foundation of the multiscale lifting numerical algorithm, which approximates the exact solution of the problems analyzed at high convergence rate. The key ingredients of the multiscale lifting algorithm are a reliable error estimation and lifting algorithm, which are described in this section.

5.1. Error estimation

Given an SGW-based finite element solution u_j and the exact solution u of the problems analyzed, the error estimation e is defined as

$$e = \left\|u - u_j\right\|_\infty = \max_{\Omega}\left|u(x) - u_j(x)\right|. \tag{26}$$

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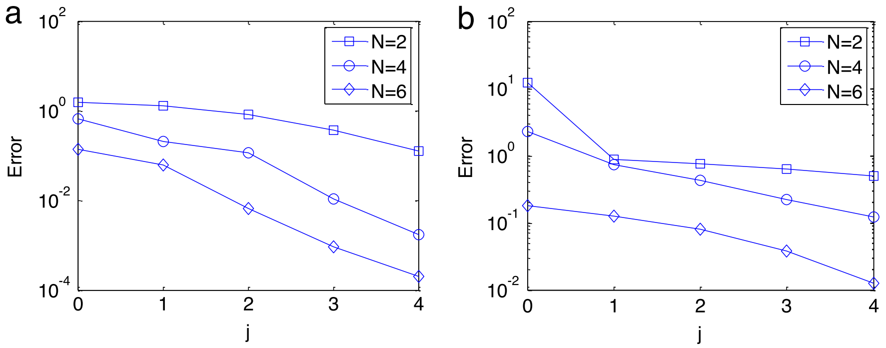

Fig. 3. Convergence of the SGW-based finite element solution with the number of levels: (a) Example 1; (b) Example 2.

Ref. [19] proves that the function $u(x)$ in space $L^2(\mathbf{R})$ can be approximated with the projection $u_j(x)$ in V_j and the projection can capture all the details of the initial function $u(x)$ as the scale j gets larger (i.e. as $j \to +\infty$), such as

$$\lim_{j\to\infty} \left\| u - u_j \right\| = 0. \tag{27}$$

Since the exact solution u cannot always be found for PDEs, we use a simple two-level error estimate in the form

$$e' = \max_{\Omega} \left| u_{j+1}(x) - u_j(x) \right| \tag{28}$$

where u_{j+1} and u_j in the neighbor approximate space, $j+1$ and j.

5.2. The multiscale lifting algorithm

Given an initial mesh on Ω and a threshold value ε, the multiscale lifting algorithm for the SGW-based finite element method is summarized as follows:

(1) Select second-generation wavelets of order N for an approximate space V_j.
(2) Compute the SGW-based finite element solution and the two-level error estimation e'.
(3) If $e' \le \varepsilon$, stop and give the answer.
(4) Lift the approximate space V_j to V_{j+1} and construct the new solving domain Ω_{j+1}; go to step (2).

6. Numerical examples

We present numerical experiments with high gradient and singularity to demonstrate the efficiency and accuracy of the SGW-based finite element method compared to those for FEM.

Example 1. Consider a typical problem with high gradient as follows:

$$-u'' = f, \quad x \in [0, 1] \tag{29}$$

with the boundary condition $u(x)|_{x=0,1} = 0$; the right-hand function is $f = 560x^6 - 200x^3$ and the exact solution $u(x) = 10x^5(1 - x^3)$.

An SGW-based finite element with order $N = 2, 4, 6$ and the multiscale lifting algorithm are used to solve this problem. Fig. 3(a) shows the convergence of the SGW-based finite element solution with the number of levels. Table 1 illustrates the comparison of the SGW-based finite element solution at each scale and the finite element solution. It can be observed that the solution by the SGW with higher order will lead to a faster convergence rate or smaller error estimation. Due to the multiresolution analysis of the SGW, the solution approximates the exact solution of the problem with high convergence rate using the dynamic lifting of the solution scale. Compared to the ten cubic finite elements (FEs), SGW(6) has shown its priority of convergence when the solution scale $j = 4$. It can be derived that the SGW with order $N = 2, 4$ will give random accurate results when the scale is lifted to higher numbers. Therefore, the SGW-based finite element method is very suitable for this problem with high gradient.

Example 2. Consider a PDE with singularities on the boundary as follows:

$$\alpha u''(x) - u(x) = 1 \quad x \in [0, 1] \tag{30}$$

with the parameter $\alpha = -0.01$, the boundary condition $u(x)|_{x=0,1} = 0$, and the exact solution $u(x) = -x - \frac{1}{-1+e^{1/\alpha}} + \frac{e^{x/\alpha}}{-1+e^{1/\alpha}}$.

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Table 1
Convergence of the SGW-based finite element solution at each scale and the finite element solution.

Type	Space				
	V_0	V_1	V_2	V_3	V_4
$N = 2$	1.5521	1.2891	0.8176	0.3587	0.1227
$N = 4$	0.6423	0.2081	0.1124	0.0106	0.0017
$N = 6$	0.1360	0.0622	0.0065	0.0009	0.0002
10 cubic FEs			0.0006		

Table 2
Convergence of the SGW-based finite element solution at each scale and the finite element solution.

Type	Space				
	V_0	V_1	V_2	V_3	V_4
$N = 2$	12.0000	0.8742	0.7581	0.6332	0.4953
$N = 4$	2.2467	0.7240	0.4227	0.2218	0.1203
$N = 6$	0.1817	0.1249	0.0794	0.0376	0.0125
10 cubic FEs			0.1084		

This problem can be solved using one SGW-based finite element with order $N = 2, 4, 6$ and the multiscale lifting algorithm. Fig. 3(b) compares the convergence of the SGW-based finite element solution with the number of levels while Table 2 shows the comparison of the SGW-based finite element solution at each scale and the finite element solution. It can seen that the multiscale computation of the singular problem using SGW(6) gives accurate results as compared with the other SGW finite elements and ten cubic FEs. Hence, the SGW-based finite element method is also suitable for the high-precision solution of singular problems.

7. Conclusions

A SGW-based finite element method is developed for solving PDEs efficiently. The advantage of SGWs over traditional wavelets is the flexible construction by selecting appropriate prediction and update coefficients according to the problems analyzed. According to the two-scale equations and normalization conditions of the SGWs, the connection coefficients on the interval are computed accurately, based on the equivalent filters. The numerical results verify the efficiency of the proposed method and the method can be applied to a wide range of PDEs and engineering problems efficiently, such as multidimensional or nonlinear equations, etc.

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References

[1] J.M. Restrepo, G.K. Leaf, Wavelet–Galerkin discretization of hyperbolic equations, J. Comput. Phys. 122 (1) (1995) 118–128.
[2] G. Beylkin, J.M. Keiser, On the adaptive numerical solution of nonlinear partial differential equations in wavelet bases, J. Comput. Phys. 132 (2) (1997) 233–259.
[3] O.V. Vasilyev, S. Paolucci, M. Sen, A multilevel wavelet collocation method for solving partial differential equations in a finite domain, J. Comput. Phys. 120 (1) (1995) 33–47.
[4] W. Cai, J.Z. Wang, Adaptive multiresolution collocation methods for initial boundary value problems of nonlinear PDEs, SIAM J. Numer. Anal. 33 (1996) 937–970.
[5] L.A. Diaz, M.T. Martin, V. Vampa, Daubechies wavelet beam and plate finite elements, Finite Elem. Anal. Des. 45 (3) (2009) 200–209.
[6] X.F. Chen, S.J. Yang, The construction of wavelet finite element and its application, Finite Elem. Anal. Des. 40 (5) (2004) 541–554.
[7] W. Sweldens, The lifting scheme: a construction of second generation wavelets, SIAM J. Math. Anal. 29 (2) (1997) 511–546.
[8] O.V. Vasilyev, C. Bowman, Second-generation wavelet collocation method for the solution of partial differential equations, J. Comput. Phys. 165 (2000) 660–693.
[9] M. Mehra, N. Kevlahan, An adaptive wavelet collocation method for the solution of partial differential equations on the sphere, J. Comput. Phys. 227 (2008) 5610–5632.
[10] J. Alam, N. Kevlahan, Simultaneous space–time adaptive wavelet solution of nonlinear parabolic differential equations, J. Comput. Phys. 214 (2006) 829–857.
[11] A. Cohen, Numerical Analysis of Wavelet Methods, Elsevier, North-Holland, 2003.
[12] W. Dahmen, C. Micchelli, Using the refinement equation for evaluating integrals of wavelets, SIAM J. Numer. Anal. 30 (1993) 507–537.
[13] G. Beylkin, On the representation of operators in bases of compactly supported wavelets, SIAM J. Numer. Anal. 6 (1992) 1716–1740.
[14] P. Monasse, V. Perrier, Orthonormal wavelet bases adapted for partial differential equations with boundary conditions, SIAM J. Math. Anal. 29 (1998) 1040–1065.
[15] E.B. Lin, X. Zhou, Connection coefficients on an interval and wavelet solutions of Burgers equation, J. Comput. Appl. Math. 135 (2001) 63–78.
[16] J. Ko, A.J. Kurdila, M.S. Pilant, A class of finite element methods based on orthonormal, compactly supported wavelets, Comput. Mech. 16 (1995) 235–244.
[17] X.F. Chen, Z.J. He, et al., A dynamic multiscale lifting computation method using Daubechies wavelet, J. Comput. Appl. Math. 188 (2006) 228–245.
[18] J.X. Ma, J.J. Xue, et al., A study of the construction and application of a Daubechies wavelet-based beam element, Finite Elem. Anal. Des. 39 (2003) 965–975.
[19] S. Mallat, A Wavelet Tour of Signal Processing, China Machine Press, Beijing, China, 2003.