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## 第一章

1-1 解：设：柴油的密度为  $\rho$ ，重度为  $\gamma$ ； $4^{\circ}\text{C}$  水的密度为  $\rho_0$ ，重度为  $\gamma_0$ 。则在同一地点的相对密度和比重为：

$$d = \frac{\rho}{\rho_0}, \quad c = \frac{\gamma}{\gamma_0}$$

$$\rho = d \times \rho_0 = 0.83 \times 1000 = 830 \text{ kg/m}^3$$

$$\gamma = c \times \gamma_0 = 0.83 \times 1000 \times 9.8 = 8134 \text{ N/m}^3$$

1-2 解： $\rho = 1.26 \times 10^6 \times 10^{-3} = 1260 \text{ kg/m}^3$

$$\gamma = \rho g = 1260 \times 9.8 = 12348 \text{ N/m}^3$$

1-3 解： $\beta_p = -\frac{\Delta V}{V} \Rightarrow \Delta p = -\frac{\Delta V}{V} = -\frac{\Delta V}{\beta_p \rho_0} E_p = 0.01 \times 1.96 \times 10^9 = 19.6 \times 10^6 \text{ N/m}^2$

1-4 解： $\beta_p = -\frac{\Delta V}{V} = -\frac{1000 \times 10^{-6}}{10^3} = 2.5 \times 10^{-9} \text{ m}^2/\text{N}$

$$E_p = \frac{1}{\beta_p} = \frac{1}{2.5 \times 10^{-9}} = 0.4 \times 10^9 \text{ N/m}^2$$

1-5 解：1) 求体积膨胀量和桶内压强

受温度增加的影响， $200 \text{ L}$  汽油的体积膨胀量为：

$$\Delta V_T = \beta_T V_0 \Delta T = 0.0006 \times 200 \times 20 = 2.4(\text{l})$$

由于容器封闭，体积不变，从而因体积膨胀量使容器内压强升高，体积压缩量等于体积膨胀量。故：

$$\Delta p = -\frac{\frac{\Delta V_T}{V_0 + \Delta V_T}}{\beta_p} = -\frac{\Delta V_T}{V_0 + \Delta V_T} E_p = \frac{2.4}{200 + 2.4} \times 14000 \times 9.8 \times 10^4 = 16.27 \times 10^6 \text{ N/m}^2$$

2) 在保证液面压强增量 0.18 个大气压下，求桶内最大能装的汽油质量。设装的汽油体积为  $V$ ，那么：体积膨胀量为：

$$\Delta V_T = \beta_T V \Delta T$$

体积压缩量为：

$$\Delta V_p = \frac{\Delta p}{E_p} (V + \Delta V_T) = \frac{\Delta p}{E_p} V (1 + \beta_T \Delta T)$$

因此, 温度升高和压强升高联合作用的结果, 应满足:

$$V_0 = V(1 + \beta_T \Delta T) - \Delta V_p = V(1 + \beta_T \Delta T) \left( 1 - \frac{\Delta p}{E_p} \right)$$
$$V = \frac{V_0}{(1 + \beta_T \Delta T) \left( 1 - \frac{\Delta p}{E_p} \right)} = \frac{200}{(1 + 0.0006 \times 20) \times \left( 1 - \frac{0.18 \times 10^5}{14000 \times 9.8 \times 10^4} \right)} = 197.63(l)$$

$$m = \rho V = 0.7 \times 1000 \times 197.63 \times 10^{-3} = 138.34(kg)$$

1-6 解: 石油的动力粘度:  $\mu = \frac{28}{100} \times 0.1 = 0.028 \text{ pa.s}$

$$\text{石油的运动粘度: } \nu = \frac{\mu}{\rho} = \frac{0.028}{1000 \times 0.9} = 3.11 \times 10^{-5} \text{ m}^2/\text{s}$$

1-7 解: 石油的运动粘度:  $\nu = \frac{40}{100} = 0.45 = 4 \times 10^{-5} \text{ m}^2/\text{s}$

$$\text{石油的动力粘度: } \mu = \rho \nu = 0.89 \times 1000 \times 4 \times 10^{-5} = 0.0356 \text{ pa.s}$$

1-8 解:  $\tau = \mu \frac{u}{\delta} = 1.147 \times \frac{0.001}{0.001} = 1147 \text{ N/m}^2$

1-9 解:  $\tau = \mu \frac{u}{\delta} = \mu \frac{u}{\frac{1}{2}(D-d)} = 0.065 \times \frac{0.5}{\frac{1}{2}(0.12 - 0.1196)} = 162.5 \text{ N/m}^2$

$$F = \pi \times d \times L \times \tau = 3.14 \times 0.1196 \times 0.14 \times 162.5 = 8.54 \text{ N}$$

## 第二章

2-4 解: 设: 测压管中空气的压强为  $p_2$ , 水银的密度为  $\rho_1$ , 水的密度为  $\rho_2$ 。在水银面建立等压面 1-1, 在测压管与容器连接处建立等压面 2-2。根据等压面理论, 有

$$p_a = \rho_1 g h + p_2 \quad (1)$$

$$p_1 + \rho_2 g (H + z) = p_2 + \rho_1 g z \quad (2)$$

由式 (1) 解出  $p_2$  后代入 (2), 整理得:

$$p_1 + \rho_2 g (H + z) = p_a - \rho_1 g h + \rho_1 g z$$

$$\begin{aligned}
 h &= \frac{p_a - p_1 - \rho_2 g H}{\rho_1 g} \\
 &= \frac{13600 \times 9.8 \times 0.745 - 1.5 \times 10^4 - 1000 \times 9.8 \times 1}{13600 \times 9.8} \\
 &= 0.559 \text{ mm (水银柱)}
 \end{aligned}$$

2-5 解：设：水银的密度为  $\rho_1$ ，水的密度为  $\rho_2$ ，油的密度为  $\rho_3$ ；  $h = 0.4$ ，  $h_1 = 1.6$ ，  $h_2 = 0.3$ ，

$h_3 = 0.5$ 。根据等压面理论，在等压面 1-1 上有：

$$\begin{aligned}
 p_0 + \rho_2 g (h_1 + h_2 + h_3) &= \rho_1 g h_3 + p_a \\
 p_0 &= \rho_1 g h_3 + p_a - \rho_2 g (h_1 + h_2 + h_3) \\
 &= 13600 \times 9.8 \times 0.5 + 1.0013 \times 10^5 - 1000 \times 9.8 \times (1.6 + 0.3 + 0.5) \\
 &= 1.39 \times 10^5 \text{ Pa}
 \end{aligned}$$

在等压面 2-2 上有：

$$\begin{aligned}
 p_0 + \rho_2 g h_1 &= \rho_1 g h + \rho_3 g H + p_a \\
 H &= \frac{\rho_2 h_1 - \rho_3 h}{\rho_3} \\
 &= \frac{1000 \times (1.6 - 0.4)}{800} \\
 &= 1.5 \text{ m}
 \end{aligned}$$

2-6 解：设：甘油的密度为  $\rho_1$ ，油的密度为  $\rho_2$ ，  $h = 0.4$ 。根据等压面理论，在等压面 1-1 上有：

$$\begin{aligned}
 p_0 + \rho_2 g (H - h) &= \rho_1 g \Delta h + p_a \\
 H &= h + \frac{\rho_1 \Delta h}{\rho_2} = 0.4 + \frac{1260 \times 0.7}{700} = 1.26 \text{ m}
 \end{aligned}$$

2-7 解：设：水银的密度为  $\rho_1$ ，油的密度为  $\rho_2$ 。根据等压面理论，当进气关 1 通气时，在等压面 1-1 上有：

$$p_0 + \rho_2 g H_1 = \rho_1 g \Delta h_1 + p_a \quad (1)$$

当进气关 2 通气时，在等压面 1-1 上有：

$$p'_0 + \rho_2 g H_2 = \rho_1 g \Delta h_2 + p'_a \quad (2)$$

式 (1) - 式 (2)，得：

$$\begin{aligned}
 \rho_2 g (H_1 - H_2) &= \rho_1 g (\Delta h_1 - \Delta h_2) \\
 \gamma_2 &= \rho_2 g = \frac{\rho_1 g (\Delta h_1 - \Delta h_2)}{H_1 - H_2} = \frac{\rho_1 g (\Delta h_1 - \Delta h_2)}{a}
 \end{aligned}$$

$$H_2 = \frac{\rho_1 g \Delta h_2}{\rho_2 g} = \frac{\rho_1 g \Delta h_2}{\gamma_2} = \frac{\Delta h_2 a}{\Delta h_1 - \Delta h_2}$$

2-8 解：设：水银的密度为  $\rho_1$ ，热水的密度为  $\rho_2$ ，锅炉内蒸汽压强为  $p_1$ ，大气压强为  $p_0$ 。

根据等压面理论，在等压面 1-1 上有：

$$p_1 = \rho_1 g h_2 + p_0 \quad (1)$$

在等压面 2-2 上有：

$$p_1 + \rho_2 g z_2 = \rho_2 g z_1 + p_0 \quad (2)$$

将式 (1) 代入 (2)，得：

$$p_0 + \rho_1 g h_2 + \rho_2 g z_2 = \rho_2 g z_1 + p_0$$

$$h_2 = z_1 - z_2 = \frac{\rho_1 h_2}{\rho_2}$$

2-9 解：设：水银的密度为  $\rho_1$ ，水的密度为  $\rho_2$ 。根据等压面理论，在等压面 1-1 上有：

$$p_A + \rho_2 g Z_A + \rho_1 g h = p_B + \rho_2 g (Z_A + h - 1)$$

$$\begin{aligned} p_A - p_B &= \rho_2 g (Z_A + h - 1) - \rho_2 g Z_A - \rho_1 g h \\ &= \rho_2 g (h - 1) - \rho_1 g h \\ &= 1000 \times 9.8 \times (0.5 - 1) - 13600 \times 9.8 \times 0.5 \\ &= -0.7154 \times 10^5 \text{ Pa} \end{aligned}$$

2-10 解：设：水银的密度为  $\rho_1$ ，油的密度为  $\rho_2$ 。根据题意，有：

$$p_A = \rho_2 g Z_A + p_2 \quad (1)$$

$$p_B = \rho_1 g (Z_A + \Delta h) + p_3 \quad (2)$$

根据等压面理论，在等压面 1-1 上有：

$$p_2 = \rho_1 g \Delta h + p_3 \quad (3)$$

将式 (3) 代入 (1)，得：

$$p_A = \rho_2 g Z_A + \rho_1 g \Delta h + p_3 \quad (4)$$

将 (4) - (2)，得：

$$\begin{aligned} p_A - p_B &= (\rho_1 - \rho_2) g \Delta h \\ &= (1000 - 920) \times 9.8 \times 0.125 \\ &= 98 \text{ Pa} \end{aligned}$$

2-11 解：设：水的密度为  $\rho_1$ ，油的密度为  $\rho_2$ 。根据题意，有：

$$p_A = \rho_1 g (Z_B + \Delta h) + p_2$$

$$p_B = \rho_1 g Z_B + \rho_2 g \Delta h + p_2$$

$$\begin{aligned} p_A - p_B &= (\rho_1 - \rho_2) g \Delta h \\ &= (1000 - 920) \times 9.8 \times 0.125 \\ &= 98 Pa \end{aligned}$$

2-12 解：设：手轮的转数为  $n$ ，则油被压缩的体积为：

$$\Delta V = -\frac{\pi}{4} d^2 n t$$

根据压缩性，有：

$$\beta_p = -\frac{\frac{\Delta V}{V}}{\frac{\Delta p}{\Delta V}} = \frac{\frac{\pi}{4} d^2 n t}{\frac{\Delta p V}{\Delta p}} \Rightarrow n = \frac{\Delta p V \beta_p}{\frac{\pi}{4} d^2 t} = \frac{250 \times 10^5 \times 300 \times 4.75 \times 10^{-10}}{\frac{\pi}{4} \times 1^2 \times 0.2} = 22.68$$

2-13 解：设：水银的密度为  $\rho_1$ ，水的密度为  $\rho_2$ 。根据等压面理论，在等压面 1-1 上有：

$$p + \rho_2 g z = \rho_1 g h + p_0 \Rightarrow p = \rho_1 g h + p_0 - \rho_2 g z$$

当测压管下移  $\Delta z$  时，根据压缩性，在等压面 1-1 上有：

$$\begin{aligned} p + \rho_2 g (z + \Delta z) &= \rho_1 g h' + p_0 \\ h' &= \frac{p + \rho_2 g (z + \Delta z) - p_0}{\rho_1 g} \\ &= \frac{\rho_1 g h + p_0 - \rho_2 g z + \rho_2 g (z + \Delta z) - p_0}{\rho_1 g} \\ &= \frac{\rho_1 g h + \rho_2 g \Delta z}{\rho_1 g} \\ &= h + \frac{\rho_2}{\rho_1} \Delta z \end{aligned}$$

2-14 解：建立坐标如图所示，根据匀加速直线运动容器中相对静止液体的等压面方程，有：

$$-\rho g z - ax = c$$

设  $x=0$  时，自由界面的  $Z$  坐标为  $Z_1$ ，则自由界面方程为：

$$z = z_1 - \frac{a}{g} x$$

设  $x=L$  时，自由界面的  $Z$  坐标为  $Z_2$ ，即：

$$z_2 = z_1 - \frac{a}{g} L \Rightarrow z_1 - z_2 = \frac{a}{g} L \Rightarrow a = \frac{g(z_1 - z_2)}{L} = \frac{gh}{L} = \frac{9.8 \times 0.05}{0.3} = 1.633 m/s^2$$

2-15 解：根据题意，容器在  $Z$  方向作匀加速运动。建立坐标如图所示，根据匀加速直线运动容器中相对静止液体的压强方程，有：

$$dp = \rho a_z dz \Rightarrow p = \rho a_z Z + c$$

当  $Z=0$  时,  $p=p_0$ , 则

$$p = \rho a_z Z + p_0$$

1) 容器以  $6 \text{m/s}^2$  匀加速向上运动时,  $a_z = 9.8 + 6 = 15.8$ , 则:

$$p = 1000 \times 15.8 \times 1 + 1 \times 10^5 = 115800 \text{Pa}$$

2) 容器以  $6 \text{m/s}^2$  匀加速向下运动时,  $a_z = 9.8 - 6 = 3.8$ , 则:

$$p = 1000 \times 3.8 \times 1 + 1 \times 10^5 = 103800 \text{Pa}$$

3) 容器匀加速自由下落时,  $a_z = 9.8 - 9.8 = 0.0$ , 则:

$$p = 1000 \times 0.0 \times 1 + 1 \times 10^5 = 100000 \text{Pa}$$

4) 容器以  $15 \text{m/s}^2$  匀加速向下运动时,  $a_z = 9.8 - 15 = -5.2$ , 则:

$$p = -1000 \times 5.2 \times 1 + 1 \times 10^5 = 94800 \text{Pa}$$

2-16 解: 建立坐标如图所示, 根据匀速旋转容器中相对静止液体的液面等压面方程, 有:

$$z = z_0 + \frac{1}{2} \frac{\omega^2}{g} r^2$$

式中  $r=0$  时, 自由界面的  $Z$  坐标为  $Z_0$ 。

1) 求转速  $n_1$

由于没有液体甩出, 旋转前后液体体积相等, 则:

$$\frac{\pi}{4} D^2 h_1 = \int_0^D 2 \times \pi \times r \times z \times dr = 2\pi \left( \frac{1}{8} Z_0 D^2 + \frac{1}{8 \times 16} \frac{\omega^2}{g} D^4 \right)$$

$$h_1 = Z_0 + \frac{1}{16} \frac{\omega^2}{g} D^2$$

$$Z_0 = h_1 - \frac{1}{16} \frac{\omega^2}{g} D^2 \quad (1)$$

当式中  $r=R$  时, 自由界面的  $Z$  坐标为  $H$ , 则:

$$H = z_0 + \frac{1}{8} \frac{\omega^2}{g} D^2 \quad (2)$$

将式 (1) 代入 (2), 得:

$$H = h_1 - \frac{1}{16} \frac{\omega^2}{g} D^2 + \frac{1}{8} \frac{\omega^2}{g} D^2$$

$$\omega = \sqrt{\frac{16(H-h_1)g}{D^2}} = \sqrt{\frac{16 \times (0.5-0.3) \times 9.8}{0.3^2}} = 18.667 \text{ rad/s}$$

$$n_1 = \frac{60\omega}{2\pi} = \frac{60 \times 18.667}{2\pi} = 178.25 \text{ r/min}$$

2)求转速  $n_2$

当转速为  $n_2$  时, 自由界面的最下端与容器底部接触,  $z_0=0$ 。因此, 自由界面方程为:

$$z = \frac{1}{2} \frac{\omega_2^2}{g} r^2$$

当式中  $r=R$  时, 自由界面的 Z 坐标为 H, 则:

$$H = \frac{1}{2} \frac{\omega_2^2}{g} R^2 \Rightarrow \omega_2 = \frac{1}{R} \sqrt{2gH} = \frac{1}{0.15} \sqrt{2 \times 9.8 \times 0.5} = 20.87 \text{ rad/s}$$

$$n_2 = \frac{60\omega_2}{2\pi} = \frac{60 \times 20.87}{2\pi} = 199.29 \text{ r/min}$$

$$h_2 = \frac{1}{16} \frac{\omega_2^2}{g} D^2 = \frac{1}{16} \frac{20.87^2}{9.8} \times 0.3^2 = 0.25 \text{ m}$$

2-17 解: 建立坐标如图所示, 根据题意, 阀门受到的液体总压力为:

$$P = \rho g \frac{1}{2} H^2 B = 1000 \times 9.8 \times \frac{1}{2} \times 1.5^2 \times 1.5 = 16537.5 \text{ N}$$

在不考虑阀门自重的情况下, 提起阀门的力 F 为:

$$F = \mu P = 0.3 \times 16537.5 = 11576.25 \text{ N}$$

2-18 解: 建立坐标如图所示。闸板为椭圆形, 长半轴  $b = \frac{1}{2 \sin 45^\circ} d = \frac{1}{\sqrt{2}} d$ , 短半轴

$a = \frac{1}{2} d$ 。根据题意, 总压力 P 为:

$$P = \pi a b \rho g y_c \sin 45^\circ = \pi \times 0.3 \times \frac{0.6}{\sqrt{2}} \times 850 \times 9.8 \times 5 = 16654 \text{ N}$$

闸板压力中心为:

$$\begin{aligned}
 y_p &= y_c + \frac{J_{cx}}{y_c S} = \frac{H}{\sin 45^\circ} + \frac{\frac{\pi}{4} ab^3}{\frac{H}{\sin 45^\circ} \pi ab} = \frac{H}{\sin 45^\circ} + \frac{\frac{1}{4} b^2}{\frac{H}{\sin 45^\circ}} = \frac{H}{\sin 45^\circ} + \frac{\frac{1}{8} d^2}{\frac{H}{\sin 45^\circ}} \\
 &= \frac{5}{\sin 45^\circ} + \frac{\frac{1}{8} \cdot 0.6^2}{\frac{5}{\sin 45^\circ}} = 7.077m
 \end{aligned}$$

在不考虑闸板自重的情况下，提起闸板的力  $F$  为：

$$F = \frac{\left( y_p - \left( \frac{H}{\sin 45^\circ} - \frac{1}{\sqrt{2}} d \right) \right) P}{d} = \frac{\left( 7.077 - \frac{5}{\sin 45^\circ} + \frac{1}{\sqrt{2}} \cdot 0.6 \right) \times 16654}{0.6} = 11941N$$

2-19 解：建立坐标如图所示。油罐端部的投影为圆形，直径为  $D=2.54m$ 。根据题意，总压力  $P$  为：

$$P = \rho g Z_c \frac{\pi}{4} D^2 = 700 \times 9.8 \times \left( \frac{2.54}{2} + 0.2 \right) \times \frac{\pi}{4} \times 2.54^2 = 51097.4N$$

压力中心为：

$$\begin{aligned}
 Z_p &= Z_c + \frac{J_{cx}}{y_c S} = \frac{D}{2} + 0.2 + \frac{\frac{64}{3} \frac{\pi}{4} D^4}{\left( \frac{D}{2} + 0.2 \right) \times \frac{\pi}{4} D^2} = \frac{D}{2} + 0.2 + \frac{\frac{16}{3} D^2}{\frac{D}{2} + 0.2} \\
 &= \frac{2.54}{2} + 0.2 + \frac{\frac{16}{3} \cdot 2.54^2}{\frac{2.54}{2} + 0.2} = 1.744m
 \end{aligned}$$

2-20 解：1) 求液面高度：

$$H = \frac{V}{\frac{\pi}{4} D^2} = \frac{1000}{\frac{\pi}{4} 16^2} = 4.9736m$$

设下圈高度为  $dz$ ，受到的压强为：

$$T = p_0 D dz + \rho g H D dz$$

2) 求下圈受到的拉应力

$$\sigma = \frac{T}{2edz} = \frac{p_0 D dz + \rho g H D dz}{2edz} = \frac{p_0 D + \rho g H D}{2e}$$

2) 求下圈壁厚  $e$

根据强度理论，有  $\sigma \leq [\sigma]$ ，则：

$$e \geq \frac{p_0 D + \rho g H D}{2[\sigma]} = \frac{0.08 \times 10^5 \times 16 + 800 \times 9.8 \times 4.9736 \times 16}{2 \times 1.176 \times 10^8} = 2.63 \times 10^{-3}m$$

2-21 解：建立坐标如图示。总压力的作用点的 z 坐标为：

$$\begin{aligned} Z_p &= Z_c + \frac{J_{cx}}{Z_c BH} \\ &= h - \frac{1}{2}H + \frac{\frac{1}{12}BH^3}{\left(h - \frac{1}{2}H\right)BH} \\ &= h - \frac{1}{2}H + \frac{\frac{1}{12}H^2}{h - \frac{1}{2}H} \end{aligned}$$

闸门能自动打开，要求

$$\begin{aligned} h - 0.4 > Z_p &= h - \frac{H}{2} + \frac{\frac{1}{12}H^2}{h - \frac{H}{2}} \\ h > \frac{\left(\frac{1}{3}H - 0.2\right)H}{\frac{1}{2}H - 0.4} &= \frac{\frac{1}{3} - 0.2}{\frac{1}{2} - 0.4} = 1.733m \end{aligned}$$

2-22 解：1) 求上半球受到的液体总压力

根据压力体原理，上半球受到的液体总压力为：

$$P = 1000 \times 9.8 \times \left[ (1+1) \times \pi \times 1^2 - \frac{2\pi}{3} \times 1^3 \right] = 41050N$$

上半球受到的液体总压力即为螺栓受到的总拉力。

2-23 解：设：油面蒸汽压为  $p_0$ ，油的密度为  $\rho$ 。建立坐标如图所示。

1) A-A 截面上的作用力

$$\begin{aligned} P_z &= p_0 DL + \rho g \left( DL \left( \frac{D}{2} + 0.2 \right) - \frac{\pi}{8} D^2 L \right) \\ &= 13600 \times 9.8 \times 0.368 \times 2.2 \times 9.6 + 720 \times 9.8 \times \left( 2.2 \times 9.6 \times (1.1 + 0.2) - \frac{\pi}{8} 2.2^2 \times 9.6 \right) \\ &= 1035873 + 64983 \\ &= 1100856N \end{aligned}$$

2) B-B 截面上的作用力

$$\begin{aligned}
 P_X &= p_0 D L + \rho g \times \left( \frac{D}{2} + 0.2 \right) \times D \times L \\
 &= 13600 \times 9.8 \times 0.368 \times 2.2 \times 9.6 + 720 \times 9.8 \times \left( \frac{2.2}{2} + 0.2 \right) \times 2.2 \times 9.6 \\
 &= 1035873 + 193730 \\
 &= 1229603 N
 \end{aligned}$$

2-24 解：根据题意，得

$$\begin{aligned}
 \rho g H \frac{\pi}{4} d_1^2 + mg &= \rho g \frac{\pi}{4} d_1^2 (H - Z) \\
 H = \frac{mg + \rho g \frac{\pi}{4} d_1^2 Z}{\rho g \frac{\pi}{4} (d_1^2 - d_2^2)} &= \frac{0.100 \times 9.8 + 750 \times 9.8 \times \frac{\pi}{4} \times 0.1^2 \times 0.15}{750 \times 9.8 \times \frac{\pi}{4} \times (0.1^2 - 0.02^2)} = 1.059 m
 \end{aligned}$$

2-25 解：根据题意，得

$$\begin{aligned}
 \rho g V + p_0 \frac{\pi}{4} d^2 + \rho g \frac{\pi}{4} d^2 H_2 &= mg + \rho g \frac{\pi}{4} d^2 H_1 + p_{AB} \frac{\pi}{4} d^2 \\
 p_0 - p_{AB} &= \frac{mg + \rho g \frac{\pi}{4} d^2 (H_2 - H_1) - \rho g V}{\frac{1}{4} \pi \times 0.1^2} \\
 &= \frac{(8500 - 1000) \times 9.8 \times \frac{4}{3} \times \pi \times \left( \frac{0.15}{2} + 1000 \times 9.8 \times \frac{1}{4} \times \pi \times 0.1^2 \times (5 - 2) \right)}{\frac{1}{4} \pi \times 0.1^2} \\
 &= 45937.47 Pa
 \end{aligned}$$

真空度为：

$$H_1 = \frac{p_0 - p_{AB}}{\rho g} = \frac{45937.47}{1000 \times 9.8} = 4.688 m$$

真空度大于 4.688m，球阀可打开。

2-26 解：根据题意，得：

$$\rho g \left( V + \frac{\pi}{4} d^2 h \right) = mg$$

$$h = \frac{m - \rho V}{\rho \frac{\pi}{4} d^2} = \frac{0.025 - 700 \times 10 \times 10^{-6}}{700 \times \frac{\pi}{4} \times 0.02^2} = 0.08185 m$$

2-27 解：设：木头的密度为  $\rho_1$ ，水的密度为  $\rho$ 。根据题意，得

$$(\rho - \rho_1) g \frac{\pi}{4} d L n = mg$$

$$n = \frac{mg}{(\rho - \rho_1)g \frac{\pi}{4} d^2 L} = \frac{10000}{(1000 - 800) \times 9.8 \times \frac{\pi}{4} \times 0.25^2 \times 10} = 10.39$$

取  $n=11$

### 第三章

补充题：

- 在任意时刻  $t$  流体质点的位置是  $x = 5t^2$ ，其迹线为双曲线  $xy = 25$ 。质点速度和加速度在  $x$  和  $y$  方向的分量是多少？
- 已知速度场  $u_x = yz + t$ ， $u_y = xz + t$ ， $u_z = xy$ 。试求当  $t=0.5$  时在  $x=2$ ， $y=1$ ， $z=3$  处流体质点的加速度。
- 已知欧拉方法描述的流速为： $u_x = xt$ ， $u_y = ty$ ， $u_z = tz$ 。试求  $t=0$  时，过点  $(100, 10)$  的流体质点的迹线。
- 流体运动由拉格朗日变数表达式为： $x = ae^t$ ， $y = be^{-t}$ ， $z = c$ 。求  $t=1$  时，位于  $(1, 1, 1)$  的流体质点及其加速度和迹线。求  $t=1$  时，通过  $(1, 1, 1)$  的流线。
- 给定二维流动： $\vec{u} = u_0 \vec{i} + v_0 \cos(kx - \alpha) \vec{j}$ ，其中  $u_0$ 、 $v_0$ 、 $k$ 、 $\alpha$  均为常数。试求在  $t=0$  时刻通过点  $(0, 0)$  的流线和迹线方程。若  $k, \alpha \rightarrow 0$ ，试比较这两条曲线。
- 已知不可压缩流动的势函数  $\phi = ax^2 + bxy - ay^2$ ，试求相应的流函数及在  $(1, 0)$  处的加速度。
- 已知不可压缩流动的流函数  $\psi = 3x^2y - y^3$ ，试求证流动为无旋流动并求相应的势函数。
- 给定拉格朗日流场： $x = ae^{-2t/k}$ ， $y = be^{t/k}$ ， $z = ce^{t/k}$ ，其中  $k$  为常数。试判断：①是否是稳态流动；②是否是不可压缩流动；③是否有旋流动。
- 已知不可压缩流体的压力场为：

$$p = 4x^3 - 2y^2 - yz^2 + 5z (N/m^2)$$

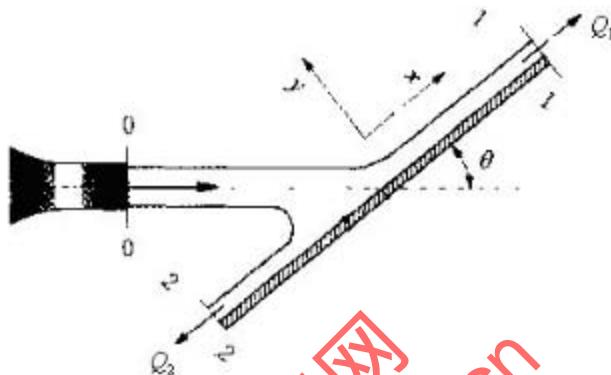
若流体的密度  $\rho = 1000 \text{ kg/m}^3$ ，则流体质点在  $(3, 1, -5)$  位置上的加速度如何？( $g = -9.8 \text{ m/s}^2$ )

- 理想不可压缩均质流体作无旋运动，已知速度势函数：

$$\phi = -\frac{2t}{\sqrt{x^2 + y^2 + z^2}}$$

在运动过程中，点  $(1, 1, 1)$  上压力总是  $p_1 = 117.7 \text{ kN/m}^2$ 。求运动开始  $20 \text{ s}$  后，点  $(4, 4, 2)$  的压力。假设质量力仅有重力。

11. 不可压缩流体平面射流冲击在一倾斜角为  $\theta = 60^\circ$  的光滑平板上, 如图所示。若喷嘴出口直径  $d=25\text{mm}$ , 喷射流量  $Q = 0.0334\text{m}^3/\text{s}$ , 试求射流沿平板两侧的分流流量  $Q_1$  和  $Q_2$ , 以及射流对平板的作用力 (不计水头损失)。



补充题答案:

1. 解: 因流体质点的连线  $xy=25$ , 得:  $y = \frac{25}{x} 5t^{-2}$

$$u_x = \frac{\partial x}{\partial t} = 10t, \quad u_x = \frac{\partial x}{\partial t^2} = 10, \quad u_y = \frac{\partial y}{\partial t} = -10t^{-3}, \quad a_y = \frac{\partial^2 y}{\partial t^2} = 30t^{-4}$$

2. 解: 根据欧拉方法, 空间点的加速度为:

$$\begin{aligned} \frac{du_x}{dt} &= \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \\ &= 1 + (yz + t) \times 0 + (xz + t)z + xy \times y \\ &= 1 + xz^2 + xy^2 + zt \end{aligned}$$

$$\begin{aligned} \frac{du_y}{dt} &= \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \\ &= 1 + (yz + t) \times z + (xz + t) \times 0 + xy \times x \\ &= 1 + yz^2 + x^2 y + zt \end{aligned}$$

$$\begin{aligned} \frac{du_z}{dt} &= \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \\ &= 0 + (yz + t) \times y + (xz + t) \times x + xy \times 0 \\ &= y^2 z + x^2 z + xt + yt \end{aligned}$$

$t=0.5$  时在  $x=2, y=1, z=3$  处流体质点的加速度为:

$$\frac{du_z}{dt} = 1 + x(z^2 + y^2) + zt = 1 + 2 \times (3^2 + 1^2) + 3 \times 0.5 = 22.5$$

$$\frac{du_y}{dt} = 1 + y(z^2 + x^2) + zt = 1 + 1 \times (3^2 + 2^2) + 3 \times 0.5 = 15.5$$

$$\frac{du_y}{dt} = z(x^2 + y^2) + (x + y)t = 3 \times (2^2 + 1^2) + (2 + 1) \times 0.5 = 16.5$$

3. 解：根据欧拉方法与拉格朗日方法的转换关系，有：

$$\frac{dx}{dt} = xt \Rightarrow \ln x = \frac{1}{2}t^2 + c \Rightarrow x = c_1 e^{\frac{1}{2}t^2}$$

$$\frac{dy}{dt} = y \Rightarrow \ln y = t + c \Rightarrow y = c_2 e^t$$

当  $t=0$  时，过点  $(100, 10)$  的流体质点的拉格朗日变数为：  $c_1 = 100$ ，  $c_2 = 10$ 。故该质点的迹线方程为：

$$x = 100e^{\frac{1}{2}t^2}, \quad y = 10e^t$$

4. 解：1) 求  $t=1$  时，位于  $(1, 1, 1)$  的流体质点及其加速度和迹线

流体质点的拉格朗日变数为  $a = \frac{1}{e} \times b + c$ ，  $c = 1$ 。该流体质点的速度和加速度为

$$u_x = \frac{\partial x}{\partial t} = ae^t = \frac{1}{e} \times e = 1, \quad a_x = \frac{\partial^2 x}{\partial t^2} = ae^t = \frac{1}{e} \times e = 1$$

$$u_y = \frac{\partial y}{\partial t} = -be^{-t} = -e \times \frac{1}{e} = -1, \quad a_y = \frac{\partial^2 y}{\partial t^2} = be^{-t} = e \times \frac{1}{e} = 1$$

$$u_z = \frac{\partial z}{\partial t} = 0, \quad a_z = \frac{\partial^2 z}{\partial t^2} = 0$$

迹线方程为：  $x = e^{t-1}$ ，  $y = e^{-t+1}$ ，  $z = 1$ ；即  $xy = 1$ 。

2) 求流线

根据拉格朗日方法与欧拉方法的转换关系，得：

$$u_x = \frac{\partial x}{\partial t} = ae^t, \quad u_y = \frac{\partial y}{\partial t} = -be^{-t}, \quad u_z = \frac{\partial z}{\partial t} = 0 \quad (1)$$

$$a = xe^{-t}, \quad b = ye^t, \quad c = z \quad (2)$$

将式 (2) 代入 (1)，得：

$$u_x = x, \quad u_y = -y, \quad u_z = 0$$

根据流线方程，有：

$$\frac{dx}{x} = \frac{dy}{-y} \Rightarrow \ln x = -\ln y + c_1 \Rightarrow xy = c$$

$t=1$  时, 流线通过(1, 1, 1)点, 则:  $c=1$ 。即流线方程:

$$xy=1$$

5. 解: 1) 求流线

$$\frac{dx}{u_0} = \frac{dy}{v_0 \cos(kx - \alpha t)} \Rightarrow \frac{1}{k} \sin(kx - \alpha t) = \frac{u_0}{v_0} y + c$$

$$y = \frac{v_0}{ku_0} \sin(kx - \alpha t) + c_1$$

当  $t=0$  时流线通过点(0, 0),  $c_1=0$ 。流线方程:

$$y = \frac{v_0}{u_0 k} \sin(kx)$$

2) 求迹线

$$\frac{dx}{dt} = u_0 \Rightarrow x = u_0 t + c_1$$

$$\frac{dy}{dt} = v_0 \cos(kx - \alpha t) = v_0 \cos(ku_0 t + kc_1 - \alpha t)$$

$$y = -\frac{v_0}{ku_0 - \alpha} \sin(ku_0 t + kc_1 - \alpha t) + c_2$$

当  $t=0$  时流体质点在点  $(0, 0)$ ,  $c_1=0$ ,  $c_2=0$ 。迹线方程:

$$x = u_0 t, \quad y = -\frac{v_0}{ku_0 - \alpha} \sin(ku_0 t - \alpha t)$$

3) 若  $k, \alpha \rightarrow 0$ , 流线为:

$$y = \frac{v_0}{u_0} x$$

迹线为:

$$x = u_0 t, \quad y = v_0 t$$

$$y = \frac{v_0}{u_0} x$$

流线与迹线重合。

6. 解: 1) 求流函数

根据势函数的性质, 有:

$$u_x = \frac{\partial \varphi}{\partial x} = 2ax + by$$

$$u_y = \frac{\partial \varphi}{\partial y} = bx - 2ay$$

根据流函数的性质，有：

$$u_x = \frac{\partial \psi}{\partial y} = 2ax + by \Rightarrow \psi = 2axy + \frac{1}{2}by^2 + c_1(x)$$

$$u_y = -\frac{\partial \psi}{\partial x} = bx - 2ay \Rightarrow -\left(2ay + \frac{\partial c_1(x)}{\partial x}\right) = bx - 2ay \Rightarrow \frac{\partial c_1(x)}{\partial x} = -bx$$

$$c_1(x) = -\frac{1}{2}bx^2 + c$$

$$\psi = 2axy + \frac{1}{2}by^2 - \frac{1}{2}bx^2 + c$$

2) 求(1, 0)处的加速度

$$\begin{aligned} \frac{du_x}{dt} &= \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} \\ &= (2ax + by) \times 2a + (bx - 2ay) \times b \\ &= 4a^2x + b^2x \\ &= 4a^2 + b^2 \end{aligned}$$

$$\begin{aligned} \frac{du_y}{dt} &= \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \\ &= (2ax + by) \times b + (bx - 2ay) \times (-2a) \\ &= b^2y + 4a^2y \\ &= 4a^2 + b^2 \end{aligned}$$

7. 解：1) 求证流动为无旋流动

根据流函数的性质，有：

$$u_x = \frac{\partial \psi}{\partial y} = 3x^2 - 3y^2$$

$$u_y = -\frac{\partial \psi}{\partial x} = -6xy$$

根据旋度，有：

$$\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = -6y - (-6y) = 0$$

旋度=0，流动为无旋流动

2) 求势函数

$$u_x = \frac{\partial \varphi}{\partial x} = 3x^2 - 3y^2 \Rightarrow \varphi = x^3 - 3xy^2 + c(y)$$

$$u_y = \frac{\partial \varphi}{\partial y} = -6xy \Rightarrow -6xy + \frac{\partial c(y)}{\partial y} = -6xy \Rightarrow c(y) = c_1$$

$$\varphi = x^3 - 3xy^2 + c_1$$

8. 解：1) 将拉格朗日方法转换为欧拉方法

$$u_x = \frac{\partial x}{\partial t} = -\frac{2a}{k}e^{-2t/k}, \quad u_y = \frac{\partial y}{\partial t} = \frac{b}{k}e^{t/k}, \quad u_z = \frac{\partial z}{\partial t} = \frac{c}{k}e^{t/k}$$

解拉格朗日变数：

$$a = xe^{2t/k}, \quad b = ye^{-t/k}, \quad c = ze^{t/k}$$

欧拉方法表示的流场：

$$u_x = -\frac{2}{k}x, \quad u_y = \frac{1}{k}y, \quad u_z = \frac{1}{k}z$$

因  $\frac{\partial u_x}{\partial t} = \frac{\partial u_y}{\partial t} = \frac{\partial u_z}{\partial t} = 0$ ，是稳态流动。

因  $\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = -\frac{2}{k} + \frac{1}{k} + \frac{1}{k} = 0$ ，是不可压流场。

因  $\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = 0, \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} = 0, \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} = 0$ ，是无旋流动。

9. 解：根据理想流体运动微分方程，有

$$\begin{aligned}\frac{du_x}{dt} &= F_x - \frac{1}{\rho} \frac{\partial p}{\partial x} \\ &= -\frac{1}{\rho} \frac{\partial}{\partial x} (4x^3 - 2y^2 - yz^2 + 5z) \\ &= -\frac{12}{\rho} x^2 \\ &= \frac{12}{1000} \times 3^2 \\ &= 0.108\end{aligned}$$

$$\begin{aligned}\frac{du_y}{dt} &= F_y - \frac{1}{\rho} \frac{\partial p}{\partial y} \\ &= -\frac{1}{\rho} \frac{\partial}{\partial y} (4x^3 - 2y^2 - yz^2 + 5z) \\ &= -\frac{1}{\rho} (-4y - z^2) \\ &= -\frac{1}{1000} (-4 - (-5)^2) \\ &= 0.029\end{aligned}$$

$$\begin{aligned}\frac{du_z}{dt} &= F_z - \frac{1}{\rho} \frac{\partial p}{\partial z} \\ &= -g - \frac{1}{\rho} \frac{\partial}{\partial z} (4x^3 - 2y^2 - yz^2 + 5z) \\ &= -g - \frac{1}{\rho} (-2yz + 5) \\ &= -9.8 - \frac{1}{1000} (-2 \times 1 \times (-5) + 5) \\ &= -9.815\end{aligned}$$

10. 解：根据势函数，有

$$u_x = \frac{\partial \varphi}{\partial x} = \frac{2tx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$u_y = \frac{\partial \varphi}{\partial y} = \frac{2ty}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$u_z = \frac{\partial \varphi}{\partial z} = \frac{2tz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

求各加速度分量:

$$\begin{aligned} \frac{du_x}{dt} &= \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \\ &= \frac{2x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{2tx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{2t(y^2 + z^2 - 2x^2)}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \\ &\quad - \frac{2ty}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{6tzy}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} - \frac{2tz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{6txz}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \\ &= \frac{2x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{t^2}{(x^2 + y^2 + z^2)^4} (4(y^2 + z^2 - 2x^2) - 12xy^2 - 12xz^2) \\ &= \frac{2x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} - \frac{8yt^2}{(x^2 + y^2 + z^2)^3} \end{aligned}$$

$$\begin{aligned} \frac{du_y}{dt} &= \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \\ &= \frac{2y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} - \frac{2tx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{6tzy}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \\ &\quad + \frac{2ty}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{2t(x^2 - 2y^2 + z^2)}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} - \frac{2tz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{6tyz}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \\ &= \frac{2y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{t^2}{(x^2 + y^2 + z^2)^4} (-12x^2y + 4(yx^2 + yz^2 - 2y^3) - 12yz^2) \\ &= \frac{2y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} - \frac{8yt^2}{(x^2 + y^2 + z^2)^3} \end{aligned}$$

$$\begin{aligned}
\frac{du_z}{dt} &= \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \\
&= \frac{2z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} - \frac{2tx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{6txz}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \\
&\quad - \frac{2ty}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{6tyz}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} - \frac{2tz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \times \frac{2t(x^2 + y^2 - 2z^2)}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \\
&= \frac{2y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{t^2}{(x^2 + y^2 + z^2)^4} (-12x^2z - 12y^2z + 4(zx^2 + zy^2 - 2z^3)) \\
&= \frac{2y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} - \frac{8zt^2}{(x^2 + y^2 + z^2)^3}
\end{aligned}$$

根据理想流体运动微分方程, 有

$$\begin{aligned}
\frac{du_z}{dt} &= F_z - \frac{1}{\rho} \frac{\partial p}{\partial x} \\
&\quad - \frac{2x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{8xt^2}{(x^2 + y^2 + z^2)^3} - \frac{1}{\rho} \frac{\partial p}{\partial t} \\
p &= \rho \left[ \frac{2}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} - \frac{2t^2}{(x^2 + y^2 + z^2)^2} + c_1(y, z, t) \right] \\
\frac{du_y}{dt} &= F_y - \frac{1}{\rho} \frac{\partial p}{\partial y} \\
&\quad - \frac{2y}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{8yt^2}{(x^2 + y^2 + z^2)^3} = \frac{\partial}{\partial y} \left[ \frac{2}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} - \frac{2t^2}{(x^2 + y^2 + z^2)^2} + c_1(y, z, t) \right]
\end{aligned}$$

$$\frac{\partial c_1(y, z, t)}{\partial y} = 0 \Rightarrow c_1(y, z, t) = c_1(z, t)$$

$$p = \rho \left[ \frac{2}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} - \frac{2t^2}{(x^2 + y^2 + z^2)^2} + c_1(z, t) \right]$$

$$\begin{aligned}
\frac{du_z}{dt} &= F_z - \frac{1}{\rho} \frac{\partial p}{\partial z} \\
&\quad - \frac{2z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{8zt^2}{(x^2 + y^2 + z^2)^3} = g + \frac{\partial}{\partial y} \left[ \frac{2}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} - \frac{2t^2}{(x^2 + y^2 + z^2)^2} + c_1(z, t) \right]
\end{aligned}$$

$$\frac{\partial c_2(z,t)}{\partial z} = -g \Rightarrow c_2(z,t) = -gz + c_3(t)$$

$$p = \rho \left[ \frac{2}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} - \frac{2t^2}{(x^2 + y^2 + z^2)^2} - gz + c_3(t) \right]$$

在运动过程中, 点(1, 1, 1)上压力总是  $p_1 = 117.7 \text{ kN/m}^2$ 。因此

$$p_1 = \rho \left[ \frac{2}{(1^2 + 1^2 + 1^2)^{\frac{1}{2}}} - \frac{2t^2}{(1^2 + 1^2 + 1^2)^2} - g + c_3(t) \right]$$

$$c_3(t) = \frac{p_1}{\rho} + g - \frac{2\sqrt{3}}{3} + \frac{2t^2}{9}$$

$$p = \rho \left[ \frac{2}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} - \frac{2t^2}{(x^2 + y^2 + z^2)^2} - g(z-1) + \frac{p_1}{\rho} - \frac{2\sqrt{3}}{3} + \frac{2t^2}{9} \right]$$

运动开始20s后, 点(4, 4, 2)的压力为?

$$\begin{aligned} p &= 1000 \times \left[ \frac{2}{(4^2 + 4^2 + 2^2)^{\frac{1}{2}}} - \frac{2 \times 20^2}{(4^2 + 4^2 + 2^2)^2} - 9.8 \times (2-1) + \frac{117.7 \times 10^3}{1000} - \frac{2\sqrt{3}}{3} + \frac{2 \times 20^2}{9} \right] \\ &= 1000 \times \left[ \frac{1}{3} - \frac{2 \times 20^2}{36^2} - 9.8 + \frac{117.7 \times 10^3}{1000} - \frac{2\sqrt{3}}{3} + \frac{2 \times 20^2}{9} \right] \\ &= 195.35 \text{ kPa} \end{aligned}$$

第二种解法:

由于流动为无旋流, 根据拉格朗日积分, 同一时刻流场中任意两点间的关系有:

$$\frac{\partial \varphi_1}{\partial t} + \frac{1}{2} u_1^2 + gz_1 + \frac{p_1}{\rho} = \frac{\partial \varphi_2}{\partial t} + \frac{1}{2} u_2^2 + gz_2 + \frac{p_2}{\rho}$$

因:

$$\frac{\partial \varphi}{\partial t} = -\frac{2}{\sqrt{x^2 + y^2 + z^2}}$$

$$u_x = \frac{\partial \varphi}{\partial x} = \frac{2tx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$u_y = \frac{\partial \varphi}{\partial y} = \frac{2ty}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$u_z = \frac{\partial \varphi}{\partial z} = \frac{2tz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

则点(1, 1, 1)的相关量为:

$$\frac{\partial \varphi_1}{\partial t} = -\frac{2}{\sqrt{1^2 + 1^2 + 1^2}} = -\frac{2}{\sqrt{3}}$$

$$u_x = u_y = u_z = \frac{2t}{(1^2 + 1^2 + 1^2)^{\frac{3}{2}}} = \frac{2t}{3\sqrt{3}}$$

$$u = \sqrt{u_x^2 + u_y^2 + u_z^2} = \frac{2t}{3\sqrt{3}} \times \sqrt{3} = \frac{2t}{3}$$

点(4, 4, 2)的相关量为:

$$\frac{\partial \varphi_2}{\partial t} = -\frac{2}{\sqrt{4^2 + 4^2 + 2^2}} = -\frac{1}{3}$$

$$u_{2x} = \frac{2 \times t \times 4}{(4^2 + 4^2 + 2^2)^{\frac{3}{2}}} = \frac{t}{27}$$

$$u_{2y} = \frac{2 \times t \times 4}{(4^2 + 4^2 + 2^2)^{\frac{3}{2}}} = \frac{t}{27}$$

$$u_{2z} = \frac{2 \times t \times 2}{(4^2 + 4^2 + 2^2)^{\frac{3}{2}}} = \frac{t}{54}$$

$$u_2 = \sqrt{u_{2x}^2 + u_{2y}^2 + u_{2z}^2} = \frac{t}{27} \times \sqrt{1+1+\frac{1}{4}} = \frac{t}{18}$$

故:

$$-\frac{2}{\sqrt{3}} + \frac{2}{9}t^2 + 9.8 \times 1 + \frac{117.7 \times 10^3}{1000} = -\frac{1}{3} + \frac{1}{2 \times 18^2}t^2 + 9.8 \times 2 + \frac{p_1}{\rho}$$

$$\frac{p_1}{\rho} = \frac{1}{3} - \frac{2}{\sqrt{3}} + \left( \frac{2}{9} - \frac{1}{2 \times 18^2} \right) \times 20^2 - 9.8 + \frac{117.7 \times 10^3}{1000} = 195.35m$$

$$p_1 = 195.35 \times 1000 = 195.35kPa$$

11. 解: 根据题意, 得:

$$v_0 = \frac{Q_0}{\frac{\pi}{4}d^2} = \frac{0.0334}{\frac{\pi}{4} \times 0.025^2} = 68.04(m/s)$$

根据伯努里方程, 有:

$$\frac{p_0}{\rho g} + \frac{v_0^2}{2g} = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} \Rightarrow v_0 = v_1$$

$$\frac{p_0}{\rho g} + \frac{v_0^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \Rightarrow v_0 = v_2$$

根据动量方程，有：

$$R_z = \rho Q_1 v_1 - \rho Q_2 v_2 - \rho Q_0 v_0 \cos \theta$$

$$R_y = -\rho Q_0 \times (-v_0 \sin \theta) = \rho Q_0 v_0 \sin \theta$$

由于在大气环境下， $R_z = 0$ ，因此

$$Q_1 - Q_2 - Q_0 \cos \theta = 0 \quad (1)$$

根据不可压缩流体的连续性方程，有：

$$Q_1 + Q_2 - Q_0 = 0 \quad (2)$$

式 (1) + (2) 得：

$$Q_1 = \frac{1}{2} Q_0 (1 + \cos \theta) = \frac{1}{2} \times 0.0334 \times (1 + \cos 60^\circ) = 0.02505 m^3/s$$

故

$$Q_2 = Q_0 - Q_1 = 0.0334 - 0.02505 = 0.00835 m^3/s$$

$$R_y = \rho Q_0 v_0 \sin \theta = 1000 \times 0.0334 \times 68.04 \times \sin 60^\circ = 1968 N$$

根据作用与反作用的关系，半径径力为：

$$F_y = -R_y = -1968 N$$

### 第三章

3-1 解：

$$\frac{du_x}{dt} = \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z}$$

$$= 0 + xy^2 \times y^2 - \frac{1}{3} y^3 \times 2xy + xy \times 0$$

$$= \frac{1}{3} xy^4$$

$$\frac{du_y}{dt} = \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z}$$

$$= 0 + xy^2 \times 0 - \frac{1}{3} y^3 \times (-y^2) + xy \times 0$$

$$= \frac{1}{3} y^5$$

$$\frac{du_z}{dt} = \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z}$$

$$= 0 + xy^2 \times y - \frac{1}{3} y^3 \times x + xy \times 0$$

$$= \frac{2}{3} xy^3$$

当  $(x, y, z) = (1, 2, 3)$  时，加速度为：

$$\frac{du_x}{dt} = \frac{1}{3} xy^4 = \frac{1}{3} \times 1 \times 2^4 = \frac{16}{3}$$

$$\frac{du_y}{dt} = \frac{1}{3} y^5 = \frac{1}{3} \times 2^5 = \frac{32}{3}$$

$$\frac{du_z}{dt} = \frac{2}{3} xy^3 = \frac{2}{3} \times 1 \times 2^3 = \frac{16}{3}$$

3-2 解：

$$\frac{du_x}{u_x} = \frac{du_y}{u_y}$$

$$\frac{dx}{\frac{B}{2\pi} \times \frac{y}{x^2 + y^2}} = \frac{dy}{\frac{B}{2\pi} \times \frac{x}{x^2 + y^2}}$$

$$\frac{dx}{y} = \frac{dy}{x}$$

$$x^2 - y^2 = C$$

3-4 解：

$$u \leq \frac{Q}{\frac{\pi}{4} d^2} \Rightarrow d \geq \sqrt{\frac{4Q}{\pi u}} = \sqrt{\frac{4 \times \frac{50 \times 1000}{3600 \times 800}}{\pi \times 0.8}} = 0.166m$$

3-5 解：由于吸入管直径大于排出管直径，根据连续性原理，排出管中液体流速大于吸入管中液体流速。设排出管中液体流速为  $u_1 = 0.7$ ，

$$u_1 \leq \frac{Q}{\frac{\pi}{4} d_1^2} \Rightarrow Q \leq \frac{\pi}{4} d_1^2 u_1 = \frac{\pi}{4} 0.1^2 \times 0.7 = 5.5 \times 10^{-3} m^3/s$$

设吸入管中液体流速为  $u_2$  为：

$$u_2 = \frac{Q}{\frac{\pi}{4} d_2^2} = \frac{5.5 \times 10^{-3}}{\frac{\pi}{4} 0.15^2} = 0.31 m/s$$

3-6 解：若液位不变，取水平出流管的中心 Z 坐标为零，则液位高度为：

$$h = \frac{0.8 \times 10^3}{1000 \times 9.8} = 8.163m$$

根据伯努里方程，有：

$$z_1 + \frac{p_1}{\rho g} + \frac{u_1^2}{2g} = p_2 + \frac{u_2^2}{2g}$$

$z_1 = h$  时， $u_1 = 0$ ，表压  $p_1 = p_0$ 。因此

$$u_2 = \sqrt{2g \left( z_1 - \frac{p_1}{\rho g} \right)} = \sqrt{2 \times 9.8 \left( 8.163 - \frac{0.6 \times 10^3}{1000 \times 9.8} \right)} = 6.324(m/s)$$

$$Q = \frac{\pi}{4} d^2 u_2 = \frac{\pi}{4} \times 0.012^2 \times 6.324 = 7.15 \times 10^{-4} (m^3/s)$$

3-7 解：取 B 容器出水管口的 Z 坐标为零，根据伯努里方程，有：

$$z_1 + \frac{p_1}{\rho g} + \frac{u_1^2}{2g} = p_2 + \frac{u_2^2}{2g}$$

$z_1 = H$  时， $u_1 = 0$ ， $p_1 = p_2$ 。因此

$$u_2 = \sqrt{2gH} = \sqrt{2 \times 9.8 \times 3} = 7.668(m/s)$$

管径为：

$$u_2 = \frac{Q}{\frac{\pi}{4} d^2} \Rightarrow d = \sqrt{\frac{4Q}{\pi u_2}} = \sqrt{\frac{4 \times \frac{100}{3600}}{\pi \times 7.668}} = 0.068(m)$$

水平管中的绝对压强由下式求得：

$$\begin{aligned} H + \frac{p_1}{\rho g} &= z + \frac{p}{\rho g} + \frac{u^2}{2g} \\ p &= \rho g \left[ H + \frac{p_1}{\rho g} - \left( z + \frac{u^2}{2g} \right) \right] \\ &= 1000 \times 9.8 \left[ 3 + \frac{1 \times 10^5}{1000 \times 9.8} - \left( 6 + \frac{7.668^2}{2 \times 9.8} \right) \right] \\ &= 0.412 \times 10^5 \end{aligned}$$

$$p - p_1 = 0.412 \times 10^5 - 1 \times 10^5 = -0.588 \times 10^5 Pa$$

3-8 解：取水管中心的Z坐标为零，根据伯努利方程，有：

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} = \frac{p_2}{\rho g} \Rightarrow u_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

根据等压面原理，有：

$$\begin{aligned} p_1 - \rho g z_A + \rho g \Delta h &= p_2 - \rho g z_A + \rho g \Delta h \\ p_2 - p_1 &= (\rho' - \rho) g \Delta h \end{aligned}$$

故

$$u_1 = \sqrt{\frac{2(\rho' - \rho) g \Delta h}{\rho}} = \sqrt{\frac{2(13600 - 1000) \times 9.8 \times 0.2}{1000}} = 7.028(m/s)$$

3-9 解：取A容器液面的Z坐标为零，根据伯努利方程，两容器油面的能量关系有：

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + h'_w$$

$u_1 = u_2$ ，因此

$$h'_w = \frac{p_1 - p_2}{\rho g} - z_2 = \frac{(3.6 - 0.3) \times 10^5}{850 \times 9.8} - 20 = 19.616m \text{ 油柱}$$

3-10 解：取水管中心的Z坐标为零，根据伯努利方程，有：

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} \Rightarrow u_2^2 - u_1^2 = \frac{2(p_1 - p_2)}{\rho}$$

设流量为Q，则：

$$u_1 = \frac{Q}{\frac{\pi}{4} D^2} \quad u_2 = \frac{Q}{\frac{\pi}{4} d^2}$$

$$Q^2 \left( \frac{16}{\pi^2 d^4} - \frac{16}{\pi^2 D^4} \right) = \frac{2(p_1 - p_2)}{\rho}$$

$$\begin{aligned} Q &= \sqrt{\left( \frac{16}{\pi^2 d^4} - \frac{16}{\pi^2 D^4} \right)} \times \frac{2(p_1 - p_2)}{\rho} \\ &= \sqrt{\left( 16(D^4 - d^4) \right)} \times \frac{2(p_1 - p_2)}{\rho} \\ &= \frac{\pi d^2}{4} \sqrt{\frac{2(p_1 - p_2)}{\rho}} \end{aligned}$$

根据等压面原理，有：

$$\begin{aligned} p_1 + \rho g z_A + \rho g \Delta h &= p_2 + \rho g z_A + \rho g \Delta h \\ p_1 - p_2 &= (\rho' - \rho) g \Delta h \end{aligned}$$

故

$$\begin{aligned} Q &= \frac{\pi d^2}{4} \sqrt{\frac{2(\rho' - \rho) \times g \times \Delta h}{\rho}} \\ &= \alpha \frac{\pi d^2}{4} \sqrt{\frac{2(\rho' - \rho) \times g \times \Delta h}{\rho}} \\ &= 0.9 \times \frac{\pi \times 0.05^2}{4} \sqrt{\frac{2 \times (13600 - 800) \times 9.8 \times 0.4}{800}} \\ &= 0.0198 (m^3/s) \end{aligned}$$

$$Q' = 3600 \rho Q = 3600 \times 800 \times 0.0198 = 57024 kg/h = 57.024 t/h$$

3-11 解：1) 求 B 管中流速

在 T 管上根据伯努利方程，有：

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} = \frac{p_3}{\rho g} + \frac{u_{3T}^2}{2g}$$

$$\frac{p_3}{\rho g} = \frac{p_1}{\rho g} + \frac{u_1^2}{2g} - \frac{u_{3T}^2}{2g}$$

$$p_3 = \rho g \left( \frac{p_1}{\rho g} + \frac{u_1^2}{2g} - \frac{u_{3T}^2}{2g} \right)$$

式中流速为：

$$u_1 = \frac{Q_T}{\frac{\pi}{4} D^2} = \frac{30 \times 10^{-3}}{\frac{\pi}{4} \times 0.16^2} = 1.492 \text{ m/s}$$

$$u_{3T} = \frac{Q_T}{\frac{\pi}{4} d^2} = \frac{30 \times 10^{-3}}{\frac{\pi}{4} \times 0.04^2} = 23.873 \text{ m/s}$$

因此

$$\begin{aligned} p_3 &= \rho g \left( \frac{p_1}{\rho g} + \frac{u_1^2}{2g} - \frac{u_{3T}^2}{2g} \right) \\ &= 900 \times 9.8 \times \left( \frac{2.4 \times 10^5}{900 \times 9.8} + \frac{1.492^2}{2 \times 9.8} - \frac{23.873^2}{2 \times 9.8} \right) \\ &= -0.1546 \times 10^5 \text{ N/m}^2 \end{aligned}$$

$p_1$  为表压强，液面表压强  $p_2 = 0$ 。在 B 管上根据伯努利方程，有：

$$\begin{aligned} \frac{p_1}{\rho_1 g} + \frac{u_1^2}{2g} &= H + \frac{p_3}{\rho_1 g} + \frac{u_{3B}^2}{2g} + h_{uB} \\ \frac{u_{3B}^2}{2g} &= \frac{p_2}{\rho_1 g} - H - h_{uB} - \frac{p_3}{\rho_1 g} \\ u_{3B} &= \sqrt{2g \left[ \frac{p_2}{\rho_1 g} - H - h_{uB} - \frac{p_3}{\rho_1 g} \right]} \\ &= \sqrt{2 \times 9.8 \times \left[ 0 - 1.5 - 0.15 - \frac{-0.1546 \times 10^5}{800 \times 9.8} \right]} \\ &= 2.512 \text{ (m/s)} \end{aligned}$$

2) 求 B 管直径

$$u_{3B} = \frac{Q_B}{\frac{\pi}{4} d_B^2} \Rightarrow d_B = \sqrt{\frac{4Q_B}{\pi u_{3B}}} = \sqrt{\frac{4 \times 0.1 \times 30 \times 10^{-3}}{\pi \times 2.512}} = 0.039 \text{ m}$$

3-12 解：根据伯努利方程，有：

$$H + \frac{p_0}{\rho g} + \frac{u_0^2}{2g} = \frac{p_0}{\rho g} + \frac{u_1^2}{2g} + \Delta h_{u1} + \Delta h_{u2}$$

则管中出口流速  $u_2 = \sqrt{2g[H - (\Delta h_{u1} + \Delta h_{u2})]} = \sqrt{2 \times 9.8 \times [3 - (0.6 + 1)]} = 5.238 \text{ (m/s)}$

$$\text{管中流量 } Q = \frac{\pi}{4} d_1^2 u_1 = \frac{\pi}{4} \times 0.01^2 \times 5.238 = 4.114 \times 10^{-5} (\text{m}^3 / \text{s})$$

$$\text{水力坡度: } i_1 = \frac{\Delta h_{w1}}{L_1} = \frac{0.6}{10} = 0.06, \quad i_2 = \frac{\Delta h_{w2}}{L_2} = \frac{1}{10} = 0.1$$

3-14 解: 根据伯努里方程, 建立两液面间的关系有:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + H = z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + \Delta h_w$$

根据意  $u_1 = u_2 = 0$ , 表压  $p_1 = p_2$ , 因此

$$H = z_2 + \Delta h_w = 2 + 27 + 1 = 30 \text{m 水柱}$$

$$\eta = \frac{\rho g H Q}{N} \Rightarrow Q = \frac{\eta N}{\rho g H} = \frac{0.9 \times 80 \times 1000}{1000 \times 9.8 \times 30} = 0.245 \text{m}^3 / \text{s}$$

根据伯努里方程, 并考虑  $u_1 = 0$ , 建立吸入液面与泵吸入口间的关系有:

$$\frac{p_1}{\rho g} = z_s + \frac{p_s}{\rho g} + \frac{u_s^2}{2g} + \Delta h_{ws} \Rightarrow \frac{p_s}{\rho g} - \frac{p_1}{\rho g} = z_s + \frac{u_s^2}{2g} + \Delta h_{ws}$$

$$\text{吸入管中流速 } u_s = \frac{Q}{\frac{\pi}{4} d^2} = \frac{0.245}{\frac{\pi}{4} \times 0.3^2} = 3.466 (\text{m/s})$$

$$\text{泵吸入口处的真空度 } \frac{p_s}{\rho g} = 2 + \frac{3.466^2}{2 \times 9.8} + 0.2 = 2.813 \text{m 水柱}, \text{ 则真空表读数为:}$$

$$-0.276 \text{at}.$$

3-15 解: 根据伯努里方程, 建立吸入液面间与压水管出口的关系有:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + H = z_1 + z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + \Delta h_w$$

根据意  $u_1 = 0$ , 表压  $p_1 = p_2$  为零。因此

$$H = z_1 + z_2 + \frac{u_2^2}{2g} + \Delta h_w = 20 + \frac{20^2}{2 \times 9.8} + 2 = 42.408 \text{m 水柱}$$

$$Q = \frac{\pi}{4} D_2^2 u = \frac{\pi}{4} \times 0.01^2 \times 20 = 1.57 \times 10^{-3} \text{m}^3 / \text{s}$$

$$N = \frac{\rho g H Q}{\eta} = \frac{1000 \times 9.8 \times 42.408 \times 1.57 \times 10^{-3}}{0.8} = 81.6 \text{W}$$

根据伯努里方程, 建立泵出口与压水管出口的关系间的:

$$\frac{p_d}{\rho g} + \frac{u_d^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + \Delta h_{w1} \Rightarrow \frac{p_d}{\rho g} - \frac{p_2}{\rho g} = z_2 + \frac{u_2^2}{2g} + \Delta h_{w1} - \frac{u_d^2}{2g}$$

$$\text{泵出口处管中流速 } u_d = \frac{Q}{\frac{\pi}{4} D^2} = \frac{1.57 \times 10^{-3}}{\frac{\pi}{4} \times 0.02^2} = 5 \text{ (m/s)}$$

$$\text{泵出口处的表压强 } \frac{p_d}{\rho g} - \frac{p_1}{\rho g} = 19 + \frac{20^2}{2 \times 9.8} + 1.7 - \frac{5^2}{2 \times 9.8} = 39.833 \text{ m 水柱}$$

3-16 解：根据伯努利方程，建立两油罐油面间的关系有：

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + H = z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + \Delta h_w$$

根据意  $u_1 = u_2 = 0$ ，因此

$$H = z_2 + \frac{p_2 - p_1}{\rho g} + \Delta h_w = 40 + \frac{(0.3 - 0.2) \times 10^5}{800 \times 9.8} + 5 = 46.276 \text{ m 油柱}$$

$$N = \rho g H Q = 800 \times 9.8 \times 46.276 \times \frac{20}{3600} = 2015 \text{ W} = 2.015 \text{ kW}$$

$$N_{\text{泵}} = \frac{N}{\eta_{\text{泵}}} = \frac{2.015}{0.8} = 2.519 \text{ kW}$$

$$N_{\text{电动机}} = \frac{N_{\text{泵}}}{\eta_{\text{电动机}}} = \frac{2.519}{0.9} = 2.519 \text{ W} = 2.8 \text{ kW}$$

3-17 解：1) 求扬程 H

根据伯努利方程，建立吸入口液面与压水管出口的关系有：

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + H = z_2 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + \Delta h_w$$

根据意  $u_1 = 0$ ， $p_1 = p_2$ ， $u_2 = 0$

$$H = z_2 + \frac{u_2^2}{2g} + \Delta h_w$$

$$Q = \frac{\eta N}{\rho g H}$$

$$u_2 = \frac{Q}{\frac{\pi}{4} D^2} = \frac{\eta N}{\frac{\pi}{4} D^2 \rho g H}$$

$$H = z_2 + \frac{1}{2g} \left( \frac{\eta N}{\frac{\pi}{4} D^2 \rho g H} \right)^2 + \Delta h_w$$

$$H^3 - (z_2 + \Delta h_{ux})H^2 - \frac{1}{2g} \left( \frac{\eta N}{\frac{\pi}{4} D^2 \rho g} \right)^2 = 0$$

$$H^3 - (3+3)H^2 - \frac{1}{2g} \left( \frac{0.9 \times 8 \times 1000}{\frac{\pi}{4} \times 0.3^2 \times 1000 \times 9.8} \right)^2 = 0$$

$$H^3 - 6H^2 - 5.512 = 0$$

解方程得:  $H=6.133\text{m}$  水柱。因此, 管中流量和流速为:

$$Q = \frac{\eta N}{\rho g H} = \frac{0.9 \times 8 \times 1000}{1000 \times 9.8 \times 6.133} = 0.12 \text{m}^3/\text{s}$$

$$u_2 = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.12}{\frac{\pi}{4} \times 0.3^2} = 1.698 \text{m/s}$$

2) 求泵入口处压强

根据伯努利方程, 并考虑  $u_1 = 0$ , 泵吸入口液面与泵吸入口间的关系有:

$$\frac{p_1}{\rho g} = z_s + \frac{p_s}{\rho g} + \frac{u_s^2}{2g} + \Delta h_{ux} \quad \frac{p_1}{\rho g} - \frac{p_s}{\rho g} = z_s + \frac{u_s^2}{2g} + \Delta h_{ux}$$

$$\text{泵吸入口处的真空度} \frac{p_s - p_1}{\rho g} = 1 + \frac{1.698}{2 \times 9.8} + 0.8 = 1.947 \text{m 水柱}$$

3-18 解: 1) 求液体受到的合外力

根据动量方程, 有:

$$F_x = \rho Q(v_{2x} - v_{1x})$$

$$F_y = \rho Q(v_{2y} - v_{1y})$$

$$\text{其中: } v_{2x} = 0, v_{1x} = \frac{Q}{\frac{\pi}{4} D^2} = \frac{100 \times 10^{-3}}{\frac{\pi}{4} \times 0.3^2} = 1.415 \text{m/s}, v_{2y} = 1.415 \text{m/s}, v_{1y} = 0. \text{ 因此}$$

$$F_x = -800 \times 100 \times 10^{-3} \times 1.415 = -113.177 \text{N}$$

$$F_y = 800 \times 100 \times 10^{-3} \times 1.415 = 113.177 \text{N}$$

2) 求弯管对液体的作用力

$$R'_x + \frac{\pi}{4} D^2 p_1 = F_x$$

$$R'_x = F_x - \frac{\pi}{4} D^2 p_1 = -113.177 - \frac{\pi}{4} \times 0.3^2 \times 2.23 \times 10^5 = -15876 \text{N}$$

$$R'_x - \frac{\pi}{4} D^2 p_2 = F_x$$

$$R'_x = F_x + \frac{\pi}{4} D^2 p_2 = 113.177 + \frac{\pi}{4} \times 0.3^2 \times 2.11 \times 10^5 = 15027.89 N$$

3) 求支座的作用力

弯管对液体的作用力与弯管受到液体的作用力为一对作用与反作用力关系,因此弯管受到液体的作用力为:

$$R_x = -R'_x = 15876 N$$

$$R_y = -R'_y = -15027.89 N$$

$$R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(15876)^2 + (-15028)^2} = 21860 N$$

支座受到弯管的作用力等于弯管受到液体的作用力。

3-19 解: 1) 求液体受到的合外力

根据动量方程,有:

$$F_x = \rho Q(v_{2x} - v_{1x}) = \rho Q(v_2 \cos 60^\circ - v_1)$$

$$F_y = \rho Q(v_{2y} - v_{1y}) = \rho Q(-v_2 \sin 60^\circ - 0)$$

$$\text{其中: } v_2 = \frac{Q}{\frac{\pi}{4} D_B^2} = \frac{0.1}{\frac{\pi}{4} \times 0.25^2} = 2.037 m/s$$

$$v_1 = \frac{Q}{\frac{\pi}{4} D_A^2} = \frac{0.1}{\frac{\pi}{4} \times 0.5^2} = 0.509 m/s$$

因此

$$F_x = 1000 \times 0.1 \times (2.037 \times \cos 60^\circ - 0.509) = 50.96 N$$

$$F_y = 1000 \times 0.1 \times (-2.037 \times \sin 60^\circ - 0) = -176.4 N$$

2) 求弯管对液体的作用力

$$\frac{p_A}{\rho g} + \frac{v_1^2}{2g} = \frac{p_B}{\rho g} + \frac{v_2^2}{2g}$$

$$p_A = p_B + \frac{\rho}{2} (v_2^2 - v_1^2) = 1.8 \times 10^5 + \frac{1000}{2} \times (2.037^2 - 0.509^2) = 1.819 \times 10^5 Pa$$

$$R'_x + \frac{\pi}{4} D_A^2 p_A - \frac{\pi}{4} D_B^2 p_B \cos 60^\circ = F_x$$

$$R'_x = F_x - \frac{\pi}{4} D_A^2 p_A + \frac{\pi}{4} D_B^2 p_B \cos 60^\circ$$

$$= 50.9 - \frac{\pi}{4} \times 0.5^2 \times 1.819 \times 10^5 + \frac{\pi}{4} \times 0.25^2 \times 1.8 \times 10^5 \times \cos 60^\circ$$

$$= -31247 N$$

$$R'_y + \frac{\pi}{4} D_B^2 p_B \sin 60^\circ = F_y$$

$$R'_y = F_y - \frac{\pi}{4} D_B^2 p_B \sin 60^\circ$$

$$= -176.4 - \frac{\pi}{4} \times 0.25^2 \times 1.8 \times 10^5 \times \sin 60^\circ = -7828.37 N$$

3) 求弯头受到液体的作用力

根据作用与反作用力关系, 有:

$$R_x = -R'_x = 31247 N$$

$$R_y = -R'_y = 7828.37 N$$

$$R = \sqrt{R_x^2 + R_y^2} = 32213 N$$

3-20 解: 1) 求液体受到的合外力

根据动量方程, 有:

$$F_x = \rho Q (v_2 - v_1)$$

其中:  $v_2 = 20 m/s$

$$Q = v_2 \frac{\pi}{4} d^2 = 20 \times \frac{\pi}{4} \times 0.01^2 = 1.57 \times 10^{-3} m^3/s$$

$$v_1 = v_2 \left( \frac{d}{D} \right)^2 = 20 \times \left( \frac{1}{5} \right)^2 = 0.8 m/s$$

因此

$$F_x = 1000 \times 1.57 \times 10^{-3} \times (-20 - (-0.8)) = -30.144 N$$

2) 求筒对液体的作用力

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + \Delta hw \quad R'_x - \frac{\pi}{4} D^2 p_1 = F_x$$

$$p_1 = p_2 + \frac{\rho}{2} (v_2^2 - v_1^2) + \rho g \Delta hw \quad R'_x = F_x + \frac{\pi}{4} D^2 p_1$$

$$= 0 + \frac{1000}{2} \times (20^2 - 0.8^2) + 1000 \times 9.8 \times 1 = -30.144 + \frac{\pi}{4} \times 0.05^2 \times 2.095 \times 10^5$$

$$= 2.095 \times 10^5 Pa \quad = 381.2 N$$

3) 求人受到的作用力

根据作用与反作用力关系, 有:

$$R_x = -R'_x = -381.2 N$$

3-21 解: 1) 求液体受到的合外力

根据动量方程, 有:

$$F_x = \rho Q (v_2 - v_1)$$

$$\text{其中: } v_2 = \frac{Q}{\frac{\pi}{4} D_2^2} = \frac{1.8}{\frac{\pi}{4} \times 1^2} = 2.292 \text{ m/s}$$

$$v_1 = v_2 \left( \frac{D_1}{D_2} \right)^2 = 2.292 \times \left( \frac{1}{1.5} \right)^2 = 1.019 \text{ m/s}$$

因此

$$F_x = 1000 \times 1.8 \times (2.292 - 1.019) = 2291.4 \text{ N}$$

2) 求筒体对液体的作用力

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$p_2 = p_1 + \frac{\rho}{2} (v_1^2 - v_2^2)$$

$$= 4 \times 10^5 + \frac{1000}{2} \times (1.019^2 - 2.292^2)$$

$$= 3.979 \times 10^5 \text{ Pa}$$

$$R'_x + \frac{\pi}{4} D_1^2 p_1 - \frac{\pi}{4} D_2^2 p_2 = F_x$$

$$R'_x = F_x - \frac{\pi}{4} D_1^2 p_1 + \frac{\pi}{4} D_2^2 p_2$$

$$= 2291.4 - \frac{\pi}{4} \times 1.5^2 \times 4 \times 10^5 + \frac{\pi}{4} \times 1^2 \times 3.979 \times 10^5$$

$$= -392057 \text{ N}$$

3) 求筒体受到液体的作用力

根据作用与反作用力关系, 有:

$$R_x = -R'_x = 392057 \text{ N}$$

筒体受到液体的作用力即为筒体对支座的作用力。

3-22 解: 1) 求体积流量

$$Q = \frac{81.6}{1000} = 81.6 \times 10^{-3} \text{ m}^3 / \text{s}$$

2) 求进出口绝对流速

$$C_{2r} = \frac{Q}{\pi d_2 b} = \frac{81.6 \times 10^{-3}}{\pi \times 0.4 \times 0.04} = 1.623 \text{ m/s}$$

$$C_2 = \frac{C_{2r}}{\cos 30^\circ} = \frac{1.623}{\cos 30^\circ} = 1.875 \text{ m/s}$$

$$C_1 = C_{br} = \frac{Q}{\pi d_1 b} = \frac{81.6 \times 10^{-3}}{\pi \times 0.2 \times 0.04} = 3.247 \text{ m/s}$$

3-23 解：1) 求叶片固定不动时受到的作用力

根据伯努里方程  $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$ ，由于  $p_1 = p_2$ ，故  $v_2 = v_1$ 。根据动量方程，液体受到的合外力有：

$$\begin{aligned} F_x &= \rho Q \left( -v_2 \cos(180^\circ - 135^\circ) - v_1 \right) \\ &= -\rho v_1^2 \frac{\pi}{4} d^2 \left( \cos(180^\circ - 135^\circ) + 1 \right) \\ &= 1000 \times 19.8^2 \times \frac{\pi}{4} \times 0.1^2 \times (\cos 45^\circ + 1) \\ &= -5256.31 \text{ N} \end{aligned}$$

根据作用与反作用力关系，有：

$$R_x = -F_x = 5256.31 \text{ N}$$

2) 求叶片运动时受到的作用力

根据相对运动动量方程，液体受到的合外力有：

$$\begin{aligned} F_x &= -\rho (v_1 - v_0)^2 \frac{\pi}{4} d^2 (\cos 45^\circ + 1) \\ &= 1000 \times (19.8 - 12)^2 \times \frac{\pi}{4} \times 0.1^2 \times (\cos 45^\circ + 1) \\ &= -815.72 \text{ N} \end{aligned}$$

根据作用与反作用力关系，有：

$$R_x = -F_x = 815.72 \text{ N}$$

## 第四章

补充题：

1. 已知一粘性流体的速度场为：  $\vec{u} = 5x^3y\vec{i} + 3xyz\vec{j} - 8xz^3\vec{k}$  (m/s) 流体动力粘性系数

$\mu = 0.144(N.s/m^2)$ ， 在点(2, 4, -6)处的  $\sigma_{yy} = -100(N/m^2)$ ， 试求该点处其它的正应力和剪应力。

2. 已知粘性不可压缩流体的速度场为：  $\vec{u} = 8x^2z\vec{i} - 6y^2z^2\vec{j} + (4yz^3 - 8xz^2)\vec{k}$  (m/s)， 流体的密度为  $930\text{kg/m}^3$ ， 动力粘度为  $\mu = 4.8(N.s/m^2)$ 。若  $z$  垂直向上， 试算出点(1, 2, 3)处的压力梯度。

1. 解：1) 求流体变形应变率

$$\frac{\partial u_x}{\partial x} = 10xy = 10 \times 2 \times 4 = 80$$

$$\frac{\partial u_y}{\partial x} = 3yz = 3 \times 4 \times (-6) = -72$$

$$\frac{\partial u_x}{\partial y} = 5x^2 = 5 \times 2^2 = 20$$

$$\frac{\partial u_x}{\partial y} = 3xz = 3 \times 2 \times (-6) = -36$$

$$\frac{\partial u_x}{\partial z} = 0$$

$$\frac{\partial u_y}{\partial z} = 3xy = 3 \times 2 \times 4 = 24$$

$$\frac{\partial u_z}{\partial x} = -8z^2 = -8 \times (-6)^2 = -288$$

$$\frac{\partial u_z}{\partial y} = 0$$

$$\frac{\partial u_z}{\partial z} = -16xz = -16 \times 2 \times (-6) = 192$$

2) 求正应力

$$\sigma_{yy} = 2\mu \frac{\partial u_y}{\partial y} - \frac{2}{3}\mu \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) - p$$

$$p = 2\mu \frac{\partial u_z}{\partial y} - \frac{2}{3}\mu \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) - \sigma_{yy}$$

$$= 2 \times 0.144 \times (-36) - \frac{2}{3} \times 0.144 \times (80 - 36 + 192) + 100$$

$$= -10.368 - 22.656 + 100$$

$$= 66.976 \text{N/m}^2$$

$$\sigma_{xx} = 2\mu \frac{\partial u_x}{\partial x} - \frac{2}{3}\mu \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) - p$$

$$= 2 \times 0.144 \times 80 - \frac{2}{3} \times 0.144 \times (80 - 36 + 192) - 66.976$$

$$= 23.04 - 22.656 - 66.976$$

$$= -66.596 N/m^2$$

$$\sigma_{zz} = 2\mu \frac{\partial u_z}{\partial z} - \frac{2}{3}\mu \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right) - p$$

$$= 2 \times 0.144 \times 192 - \frac{2}{3} \times 0.144 \times (80 - 36 + 192) - 66.976$$

$$= 55.296 - 22.656 - 66.976$$

$$= -34.336 N/m^2$$

2) 求切应力

$$\tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right) = 0.144 \times (-71 + 20) = -7.488 N/m^2$$

$$\tau_{yz} = \tau_{zy} = \mu \left( \frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} \right) = 0.144 \times (0 + 24) = -3.456 N/m^2$$

$$\tau_{xz} = \tau_{zx} = \mu \left( \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) = 0.144 \times (0 - 288) = -41.472 N/m^2$$

2. 解: 1) 求各速度分量

$$u_x = 8x^2z = 8 \times 1^2 \times 3 = 24 \quad u_y = -6y^2z^2 = -6 \times 2^2 \times 3^2 = -216$$

$$u_z = 4yz^3 - 8xz^3 = 4 \times 2 \times 3^3 - 8 \times 1 \times 3^3 = 144$$

2) 求速度的偏导数

$$\frac{\partial u_x}{\partial x} = 16xz = 16 \times 1 \times 3 = 48$$

$$\frac{\partial^2 u_x}{\partial x^2} = 16z = 16 \times 1 \times 3 = 48$$

$$\frac{\partial u_x}{\partial y} = 0$$

$$\frac{\partial^2 u_x}{\partial y^2} = 0$$

$$\frac{\partial u_x}{\partial z} = 8x^2 = 8 \times 1 = 8$$

$$\frac{\partial^2 u_x}{\partial z^2} = 0$$

$$\frac{\partial u_y}{\partial x} = 0$$

$$\frac{\partial^2 u_y}{\partial x^2} = 0$$

$$\frac{\partial u_y}{\partial y} = -12yz^2 = -12 \times 2 \times 3^2 = -216$$

$$\frac{\partial^2 u_y}{\partial y^2} = -12z^2 = -12 \times 3^2 = -108$$

$$\frac{\partial u_y}{\partial z} = -12y^2z = -12 \times 2^2 \times 3 = -144$$

$$\frac{\partial^2 u_y}{\partial z^2} = -12y^2 = -12 \times 2^2 = -48$$

$$\frac{\partial u_z}{\partial x} = -8z^2 = -8 \times 3^2 = -72$$

$$\frac{\partial^2 u_z}{\partial x^2} = 0$$

$$\frac{\partial u_z}{\partial y} = 4z^3 = 4 \times 3^3 = 108$$

$$\frac{\partial^2 u_z}{\partial y^2} = 0$$

$$\begin{aligned}\frac{\partial u_z}{\partial z} &= 12yz^2 - 16xz \\ &= 12 \times 2 \times 3^2 - 16 \times 1 \times 3 = 168\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 u_z}{\partial z^2} &= 24yz - 16x \\ &= 24 \times 2 \times 3 - 16 \times 1 = 128\end{aligned}$$

3) 求各加速度分量

$$\begin{aligned}\frac{du_z}{dt} &= \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \\ &= 0 + 24 \times 48 + (-216) \times 0 + 144 \times 8 \\ &= 2304\end{aligned}$$

$$\begin{aligned}\frac{du_y}{dt} &= \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \\ &= 0 + 24 \times 0 + (-216) \times (-216) + 144 \times (-144) \\ &= 25920\end{aligned}$$

$$\begin{aligned}\frac{du_x}{dt} &= \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \\ &= 0 + 24 \times (-72) + (-216) \times 108 + 144 \times 168 \\ &= -864\end{aligned}$$

4) 求压力梯度

根据常黏度不可压缩粘性流体运动微分方程, 有:

$$\rho \frac{du_z}{dt} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right)$$

$$\frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) - \rho \frac{du_z}{dt}$$

$$= 4.8 \times (48 + 0 + 0) - 930 \times 2304$$

$$= -2142490 \frac{N}{m^3}$$

$$\rho \frac{du_y}{dt} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right)$$

$$\frac{\partial p}{\partial y} = \mu \left( \frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) - \rho \frac{du_y}{dt}$$

$$= 4.8 \times (0 - 108 - 48) - 930 \times 25920$$

$$= -24106349 \frac{N}{m^3}$$

$$\rho \frac{du_z}{dt} = -\rho g - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right)$$

$$\begin{aligned}\frac{\partial p}{\partial z} &= -\rho g + \mu \left( \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) - \rho \frac{du_z}{dt} \\ &= -930 \times 9.8 + 4.8 \times (0 + 0 + 128) - 930 \times (-864) \\ &= 795020 \frac{N}{m^3}\end{aligned}$$

4-1. 解：

$$Re = \frac{\rho V D}{\mu} \Rightarrow V = \frac{\mu Re}{\rho D} = \nu \frac{Re}{D} = 6.7 \times 10^{-6} \times \frac{2000}{0.1} = 0.134(m/s)$$

$$Q = \rho \frac{\pi}{4} D^2 V \times 3600 = 0.85 \times \frac{\pi}{4} \times 0.1^2 \times 0.134 \times 3600 = 3.22(t/h)$$

$$4-2. \text{解: } V = \frac{Q}{\frac{\pi}{4} d^2} = \frac{4 \times 10^{-3}}{\frac{\pi}{4} \times 0.04^2} = 3.183(m/s)$$

查图温度  $20^{\circ}\text{C}$  时,  $\nu = 1.88 \text{st}$ , 则

$$Re = \frac{\rho V D}{\mu} = \frac{\nu D}{\nu} = \frac{3.183 \times 0.04}{1.88 \times 10^{-4}} = 707 \text{湍流}$$

查图温度  $40^{\circ}\text{C}$  时,  $\nu = 0.55 \text{st}$ , 则

$$Re = \frac{\rho V D}{\mu} = \frac{\nu D}{\nu} = \frac{3.183 \times 0.04}{0.55 \times 10^{-4}} = 2546 \text{湍流}$$

当流动为层流临界状态时, 运动粘度为:

$$\nu = \frac{VD}{Re} = \frac{3.183 \times 0.04}{2000} = 6.366 \times 10^{-5} \text{m}^2/\text{s} = 0.6366 \text{cm}^2/\text{s}$$

查图得, 温度为  $35^{\circ}\text{C}$ 。

$$4-3. \text{解: } Re = \frac{\rho V D}{\mu} \Rightarrow D = \frac{\mu Re}{\rho V} = \frac{45 \times 10^{-3} \times 2000}{900 \times 1} = 0.1m$$

$$4-4. \text{解: } V = \frac{Q}{\frac{\pi}{4} d^2} = \frac{1.66 \times 10^{-3}}{\frac{\pi}{4} \times 0.1^2} = 0.211(m/s)$$

$$Re = \frac{\rho V D}{\mu} \Rightarrow \mu = \frac{\rho V D}{Re} = \frac{880 \times 0.211 \times 0.1}{2000} = 9.284 \times 10^{-3} \text{Pa.s} = 9.284 \text{cP}$$

4-10. 解: 由于为层流, 流体仅沿 X 方向有流动。根据连续性方程, 有

$$\frac{\partial u_x}{\partial x} = 0 \Rightarrow u_x = C(y)$$

根据运动微分方程和已知条件，有：

$$\rho g + \mu \frac{\partial^2 u_z}{\partial y^2} = 0$$

$$\frac{\partial^2 u_z}{\partial y^2} = -\frac{\rho g}{\mu}$$

$$u_z = -\frac{\rho g}{2\mu} y^2 + c_1 y + c_2$$

当  $y = 0$ ,  $u_z = 0$ , 故  $c_2 = 0$ ; 当  $y = \delta$ ,  $u_z = 0$ , 故  $\frac{\partial u_z}{\partial y} = 0$ , 即  $c_1 = \frac{\rho g}{\mu} \delta$ 。因此

$$u_z = \frac{\rho g}{\mu} \left( \delta y - \frac{1}{2} y^2 \right) \Rightarrow u_z = \frac{\rho g}{\mu} \delta^2 \left( \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^2 \right)$$

4-11. 解：根据圆管层流的速度分布，最大速度在  $r=0$  处，且平均速度为最大速度的二分之一，故

$$V = \frac{1}{2} u_{\max} = \frac{1}{2} \times 4 = 2 \text{ m/s}$$

$$u_{\max} = \frac{\Delta p}{4\mu L} R^2$$

$$\frac{\Delta p}{4\mu L} = \frac{u}{R^2}$$

因此，圆管内层流的速度分布有

$$u = \frac{\Delta p}{4\mu L} \left( R^2 - r^2 \right)$$

当  $u = V$  时，即

$$\frac{1}{2} u_{\max} = \frac{u_{\max}}{R^2} \left( R^2 - r^2 \right)$$

$$r = \frac{1}{\sqrt{2}} R = \frac{1}{\sqrt{2}} \times 200 = 141.42 \text{ mm}$$

4-12. 解：圆管内层流的速度分布有

$$u = \frac{\Delta p}{4\mu L} \left( a^2 - r^2 \right)$$

$$\frac{du}{dr} = -\frac{\Delta p}{2\mu L} r$$

$$\tau = -\mu \frac{du}{dr} = \frac{\Delta p}{2L} r$$

圆管内层流的平均速度为最大速度的二分之一，即

$$V = \frac{1}{2} u_{\max} = \frac{\Delta p a^2}{8\mu L} \Rightarrow \frac{\Delta p}{L} = \frac{8\mu V}{a^2}$$

当  $r = a$  时，最大切应力为：

$$\tau_{\max} = \frac{4\mu V}{a^2} a = \frac{4\mu V}{a}$$

4-13. 解：1) 圆管内平均速度

$$Q = \frac{200 \times 10^3}{3600 \times 900} = 0.0617 \text{ m}^3/\text{s}$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.0617}{\frac{\pi}{4} \times 0.3^2} = 0.873 \text{ m/s}$$

2) 求沿程损失

$$Re_1 = \frac{\rho V D}{\mu_1} = \frac{V D}{\nu_1} = \frac{0.873 \times 0.3}{1.5 \times 10^{-4}} = 104.76$$

$$Re_2 = \frac{\rho V D}{\mu_2} = \frac{V D}{\nu_2} = \frac{0.873 \times 0.3}{1.5 \times 10^{-4}} = 1746$$

因雷诺数小于 2000，流动为层流，故

$$\lambda_1 = \frac{64}{Re_1} = \frac{64}{104.76} = 0.611$$

$$\lambda_2 = \frac{64}{Re_2} = \frac{64}{1746} = 0.0367$$

沿程损失有：

$$h_{f1} = \lambda_1 \frac{L V^2}{D 2g} = 0.611 \times \frac{5000}{0.3} \times \frac{0.873^2}{2 \times 9.8} = 395.97 \text{ m}$$

$$h_{f2} = \lambda_2 \frac{L V^2}{D 2g} = 0.0367 \times \frac{5000}{0.3} \times \frac{0.873^2}{2 \times 9.8} = 23.784 \text{ m}$$

$$\frac{h_{f1} - h_{f2}}{h_{f1}} = \frac{395.97 - 23.784}{395.97} = 0.0367 \times \frac{5000}{0.3} \times \frac{0.873^2}{2 \times 9.8} = 0.94 = 94\%$$

4-14. 解：

$$\begin{aligned} Q &= \int u 2\pi(R-y) dy \\ &= \int 2 \times 8.74 \times \pi \times U_* \times \left( \frac{\rho U_* y}{\mu} \right)^{\frac{1}{7}} (R-y) dy \\ &= \int 2 \times 8.74 \times \pi \times U_* \times R \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{7}} \left( \frac{y}{R} \right)^{\frac{1}{7}} \left( 1 - \frac{y}{R} \right) d \frac{y}{R} \\ &= 2 \times 8.74 \times \pi \times U_* \times R \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{7}} \left[ \frac{7}{8} \left( \frac{y}{R} \right)^{\frac{8}{7}} - \frac{7}{15} \left( \frac{y}{R} \right)^{\frac{15}{7}} \right] \\ &= 7.138 \times \pi \times R \times U_* \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{7}} \\ u_m &= \frac{Q}{\pi R^2} = \frac{7.138 \times \pi \times R^2 \times U_* \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{7}}}{\pi R^2} = 7.138 \times U_* \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{7}} \end{aligned}$$

$$\frac{u_m}{U_*} = 7.138 \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{7}}$$

4-15. 解：

$$\begin{aligned} Q &= \int_0^R u \times 2\pi r dr \\ &= \int_0^R 2\pi u_{\max} \left( 1 - \frac{r}{R} \right)^{\frac{1}{n}} r dr \\ &= \int_0^R 2\pi R^2 u_{\max} \left( 1 - \frac{r}{R} \right)^{\frac{1}{n}} \frac{r}{R} d \frac{r}{R} \\ &= \int_0^1 2\pi R^2 u_{\max} (1-y)^{\frac{1}{n}} y dy \\ &= 2\pi R^2 u_{\max} \left[ -\frac{n}{1+n} (1-y)^{\frac{1+n}{n}} y - \frac{n}{n+1} \times \frac{n}{1+2n} (1-y)^{\frac{1+2n}{n}} \right]_0^1 \\ &= 2\pi R^2 u_{\max} \frac{n^2}{(n+1)(1+2n)} \end{aligned}$$

$$u_m = \frac{Q}{\pi R^2} = \frac{2\pi R^2 u_{\max} \frac{n^2}{(n+1)(1+2n)}}{\pi R^2} = 2u_{\max} \frac{n^2}{(n+1)(1+2n)}$$

$$\frac{u_m}{u_{\max}} = \frac{2n^2}{(n+1)(1+2n)}$$

4-16. 解：

$$\frac{u_m}{U_*} = 7.138 \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{2}}$$

$$\lambda = 8 \left( \frac{U_*}{u_m} \right)^2 \Rightarrow \frac{u_m}{U_*} = \frac{2\sqrt{2}}{\sqrt{\lambda}}$$

$$7.138 \times \left( \frac{\rho U_* R}{\mu} \right)^{\frac{1}{2}} = \frac{2\sqrt{2}}{\sqrt{\lambda}}$$

$$7.138 \times \left( \frac{\rho D u_m}{2\mu} \times \frac{U_*}{u_m} \right)^{\frac{1}{2}} = \frac{2\sqrt{2}}{\sqrt{\lambda}}$$

$$7.1385 \times \left( \text{Re} \times \frac{\sqrt{\lambda}}{4\sqrt{2}} \right)^{\frac{1}{2}} = \frac{2\sqrt{2}}{\sqrt{\lambda}}$$

$$\lambda^{\frac{4}{7}} = \frac{2\sqrt{2} \times (4\sqrt{2})^{\frac{1}{2}}}{7.138 \times (\text{Re})^{\frac{1}{2}}} = \frac{0.50755}{(\text{Re})^{\frac{1}{2}}}$$

$$\lambda = \frac{(0.50755)^{\frac{7}{4}}}{(\text{Re})^{\frac{1}{4}}} = \frac{0.3052}{(\text{Re})^{\frac{1}{4}}}$$

4-17. 解：1) 求沿程阻力系数

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{50 \times 10^{-3}}{\frac{\pi}{4} \times 0.15^2} = 2.829 \text{ m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\rho D}{\nu} = \frac{2.829 \times 0.15}{10 \times 10^{-6}} = 42435$$

因  $\text{Re} < 10^5$ ，在水力光滑区，故

$$\lambda = \frac{0.3164}{(\text{Re})^{\frac{1}{4}}} = \frac{0.3164}{(42435)^{\frac{1}{4}}} = 0.022$$

2) 求第一种情况的压降

$$h_f = \lambda \frac{L V^2}{D 2g} = 0.022 \times \frac{1000}{0.15} \times \frac{2.829^2}{2 \times 9.8} = 59.888 \text{ 油柱}$$

$$\Delta p = \rho g h_f = 9.8 \times 800 \times 59.888 = 469523 \text{ Pa} = 4.695 \text{ at}$$

3) 求第二种情况的起点压头

$$h_f = \lambda \frac{L}{D} \frac{V^2}{2g} = 0.022 \times \frac{10000}{0.15} \times \frac{2.829^2}{2 \times 9.8} = 598.88 \text{油柱}$$

根据伯努里方程，有：

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

由于管径相同， $V_1 = V_2$ ，故

$$\frac{p_1}{\rho g} = z_2 - z_1 + \frac{p_2}{\rho g} + h_f = 20 + \frac{1 \times 10^5}{800 \times 9.8} + 598.88 = 631.635 \text{油柱}$$

4-18. 解：根据伯努里方程，有：

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

由于管径相同， $V_1 = V_2$ ，不计高差，故

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{(17.6 - 1) \times 10^5}{860 \times 9.8} = 196.963 \text{油柱}$$

因流量未知，设  $V=1.25$ ，因此

$$Re = \frac{\rho V D}{\mu} = \frac{\rho V D}{\mu} = \frac{1.25 \times 0.25}{0.3 \times 10^{-4}} = 10416.67$$

因  $Re < 10^5$ ，在水力光滑区，故

$$\lambda = \frac{0.3164}{(Re)^{\frac{1}{4}}} = \frac{0.3164}{(10416.67)^{\frac{1}{4}}} = 0.03132$$

$$h_f = \lambda \frac{L}{D} \frac{V^2}{2g} = 0.03138 \times \frac{20000}{0.25} \times \frac{4^2}{2 \times 9.8} = 199.74 > 196.963$$

再设  $V=1.24$ ，因此

$$Re = \frac{\rho V D}{\mu} = \frac{\rho V D}{\mu} = \frac{1.24 \times 0.25}{0.3 \times 10^{-4}} = 10333$$

因  $Re < 10^5$ ，在水力光滑区，故

$$\lambda = \frac{0.3164}{(Re)^{\frac{1}{4}}} = \frac{0.3164}{(10333)^{\frac{1}{4}}} = 0.03138$$

$$h_f = \lambda \frac{L}{D} \frac{V^2}{2g} = 0.03138 \times \frac{20000}{0.25} \times \frac{3^2}{2 \times 9.8} = 196.95 = 196.96$$

故管中流量为：

$$Q = \frac{\pi}{4} D^2 V = \frac{\pi}{4} \times 0.25^2 \times 1.24 = 0.0609 m^3 / s$$

4-19. 解：1)计算测量的阻力系数  $\lambda$

根据伯努里方程，有：

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

由于管径相同，  $V_1 = V_2$ ， 故

$$h_f = z_1 - z_2 + \frac{p_1 - p_2}{\rho g} = 52 - 27 + \frac{(15 - 2) \times 10^5}{820 \times 9.8} = 186.772 \text{ 油柱}$$

$$Q = \frac{5500 \times 1000}{820 \times 24 \times 3600} = 0.07763 m^3 / s$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.07763}{\frac{\pi}{4} \times 0.305^2} = 1.063 m / s$$

$$h_f = \lambda \frac{L V^2}{D 2g} \Rightarrow \lambda = \frac{h_f}{\frac{L V^2}{D 2g}} = \frac{186.772}{\frac{50000 \times 1.063^2}{0.305 \times 2 \times 9.8}} = 0.0198$$

2)按经验公式计算的阻力系数

$$Re = \frac{\rho V D}{\mu} = \frac{\rho V D}{\nu} = \frac{1.063 \times 0.305}{2.5 \times 10^{-5}} = 129686$$

$$3000 < Re < \frac{59.7}{\epsilon^{\frac{8}{5}}} = \frac{59.7}{\left(\frac{2\Delta}{D}\right)^{\frac{8}{5}}} = \frac{59.7}{\left(\frac{2 \times 0.15}{305}\right)^{\frac{8}{5}}} = 163211$$

$$\lambda_i = \frac{0.3164}{(Re)^{\frac{1}{4}}} = \frac{0.3164}{(129686)^{\frac{1}{4}}} = 0.0167$$

可见  $\lambda_i < \lambda$ 。

4-20. 证明

$$i = \frac{\lambda V^2}{2gd} \Rightarrow V = \sqrt{\frac{2gd}{\lambda} i}$$

$$Q = AV = A \sqrt{\frac{2gd}{\lambda} i} \quad (1)$$

式(1)中  $A = \frac{\pi}{4} d^2$ ， 故

$$Q = \frac{\pi}{4} d^2 \sqrt{\frac{2gd}{\lambda}} i = K \sqrt{i}$$

$$\text{其中: } K = \frac{\pi}{4} d^2 \sqrt{\frac{2gd}{\lambda}} = \frac{\pi}{4} \sqrt{2g} \sqrt{\frac{d^5}{\lambda}} = \frac{\pi}{4} \times \sqrt{2 \times 9.8} \sqrt{\frac{d^5}{\lambda}} = 3.48 \sqrt{\frac{d^5}{\lambda}}$$

$$\text{水力半径 } R = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{1}{4} d, \text{ 将 } d = 4R \text{ 代入式 (1), 得}$$

$$Q = A \sqrt{\frac{8gR}{\lambda}} i = \sqrt{\frac{8g}{\lambda}} A \sqrt{Ri} = CA \sqrt{Ri}$$

$$\text{其中: } C = \sqrt{\frac{8g}{\lambda}} i.$$

4-21. 解: 查附录 1, 20°C 的清水  $\rho = 998.2 \text{ kg/m}^3$ ,  $\mu = 1.005 \times 10^{-3} \text{ Pa.s}$ .

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{100}{\frac{\pi}{4} \times 0.2^2 \times 600} = 0.884 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{998.2 \times 0.884 \times 0.2}{1.005 \times 10^{-3}} = 175603$$

$$10^5 < Re < 3 \times 10^5$$

$$\lambda = 0.0032 + \frac{0.221}{(Re)^{0.237}} = 0.0032 + \frac{0.221}{(175603)^{0.237}} = 0.0158$$

$$i = \frac{h_f}{L} = \lambda \frac{1}{D} \frac{V}{2g} = \frac{0.0158}{0.2} \times \frac{0.884^2}{2 \times 9.8} = 3.15 \times 10^{-3}$$

$$h_f = Li = 1800 \times 3.15 \times 10^{-3} = 5.67 \text{ m}$$

4-22. 解: 查表 4-6, 铸铁管的粗糙度  $\Delta = \frac{0.5 + 0.85}{2} = 0.675 \text{ mm}$ . 查附录 1, 20°C 的清水  $\rho = 998.2 \text{ kg/m}^3$ ,  $\mu = 1.005 \times 10^{-3} \text{ Pa.s}$ .

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{7.5 \times 10^{-3}}{\frac{\pi}{4} \times 0.1^2} = 0.955 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{998.2 \times 0.955 \times 0.1}{1.005 \times 10^{-3}} = 94846$$

根据  $26.98\left(\frac{D}{\Delta}\right)^{\frac{8}{7}} < \text{Re} < 4160\left(\frac{D}{2\Delta}\right)^{0.85}$  得：

$$26.98\left(\frac{100}{0.675}\right)^{\frac{8}{7}} < \text{Re} < 4160\left(\frac{100}{2 \times 0.675}\right)^{0.85}$$

$$8163 < \text{Re} < 161550$$

故采用下式计算沿程阻力系数：

$$\frac{1}{\sqrt{\lambda}} = 1.74 - 0.87 \ln\left(\frac{\Delta}{R} + \frac{18.7}{\text{Re}\sqrt{\lambda}}\right)$$

解得  $\lambda = 0.034$

$$i = \frac{h_f}{L} = \lambda \frac{1}{D} \frac{V^2}{2g} = \frac{0.034}{0.1} \times \frac{0.955^2}{2 \times 9.8} = 1.58 \times 10^{-2}$$

$$h_f = Li = 2000 \times 1.58 \times 10^{-2} = 31.64 \text{m}$$

4-23. 解： 1) 一般计算

$$Q = \frac{108 \times 1000}{750 \times 3600} = 0.04 \text{m}^3/\text{s}$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.04}{\frac{\pi}{4} \times 0.2^2} = 1.273 \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{1000 \times 1.273 \times 0.2}{4 \times 10^{-6}} = 63650$$

2) 求吸入管的局部能量损失  $\sum h_j$

查表 4-8, 带保险活门处的局部阻力系数  $\xi_1 = 0.9$ , 弯头局部阻力系数  $\xi_2 = 0.5$ , 阀门

局部阻力系数  $\xi_3 = 0.4$ , 透明油品过滤器局部阻力系数  $\xi_4 = 1.7$ .

$$\begin{aligned}\sum h_j &= (\xi_1 + 2\xi_2 + 2\xi_3 + \xi_4) \frac{V^2}{2g} \\ &= (0.9 + 2 \times 0.5 + 2 \times 0.4 + 1.7) \times \frac{1.273^2}{2 \times 9.8} \\ &= 0.364 \text{m}\end{aligned}$$

3) 求吸入管的沿程能量损失  $h_f$

因  $\text{Re} < 10^5$ , 在水力光滑区, 故

$$\lambda = \frac{0.3164}{(Re)^{\frac{1}{4}}} = \frac{0.3164}{(63650)^{\frac{1}{4}}} = 0.0199$$

$$h_f = \lambda \frac{L V^2}{D 2g} = 0.0199 \times \frac{20}{0.2} \times \frac{1.273^2}{2 \times 9.8} = 0.1647m$$

4) 求吸入管的总能量损失  $h_w$

$$h_w = h_f + \sum h_j = 0.364 + 0.1647 = 0.529m \text{ 油柱}$$

5) 求真空表的读数  $p_v$

在液面和真空表处运用伯努里方程, 有:

$$z_0 + \frac{p_0}{\rho g} + \frac{V_0^2}{2g} = z_v + \frac{p_v}{\rho g} + \frac{V_v^2}{2g} + h_w$$

根据题意  $V_0 = 0$ ,  $V_v = V$ , 压强取表压强, 故

$$\begin{aligned} p_v &= \rho g \left( z_0 - z_v + \frac{p_0}{\rho g} - \frac{V_0^2}{2g} - h_w \right) \\ &= 750 \times 9.8 \times \left( -4 - \frac{0.1 \times 10^5}{750 \times 9.8} - \frac{1.273^2}{2 \times 9.8} - 0.529 \right) \\ &= -23896 Pa \\ &= -0.239 atm \end{aligned}$$

5) 求泵的额定功率  $N$

在液面和泵的压力表处运用伯努里方程, 有:

$$z_0 + \frac{p_0}{\rho g} + \frac{V_0^2}{2g} + H = z_p + \frac{p_p}{\rho g} + \frac{V_p^2}{2g} + h_w$$

$$\begin{aligned} H &= z_p - z_0 + \frac{p_p - p_0}{\rho g} + h_w + \frac{V_p^2}{2g} \\ &= 4 + \frac{(7.25 - 0.1) \times 10^5}{750 \times 9.8} + \frac{1.273^2}{2 \times 9.8} \\ &= 101.36m \end{aligned}$$

$$N = \frac{\rho g H Q}{\eta} = \frac{750 \times 9.8 \times 101.36 \times 0.04}{0.8} = 29800W = 29.8kW$$

## 第五章

5-1. 解: 1) 一般计算

$$Q = \frac{200 \times 1000}{880 \times 3600} = 0.06313 \text{ m}^3/\text{s}$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.06313}{\frac{\pi}{4} \times 0.257^2} = 1.217 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{1.217 \times 0.257}{0.276 \times 10^{-4}} = 1332$$

2) 求沿程能量损失  $h_f$  和水力坡度

因  $Re < 10^5$ , 在水力光滑区, 故

$$\lambda = \frac{0.3164}{(Re)^{1/4}} = \frac{0.3164}{(1332)^{1/4}} = 0.03067$$

$$h_f = \lambda \frac{L V^2}{D 2g} = 0.03067 \times \frac{50000}{0.257} \times \frac{1.217^2}{2 \times 9.8} = 450.84 \text{ m}$$

$$\text{水力坡度 } i = \frac{h_f}{L} = \frac{450.84}{50000} = 9.017 \times 10^{-3}$$

3) 求压降

根据伯努里方程, 有:

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

根据题意,  $V_1 = V_2 = V$ , 故压降为:

$$\begin{aligned} p_1 - p_2 &= \rho g (z_2 - z_1 + h_f) \\ &= 880 \times 9.8 \times (84 - 45 + 450.84) \\ &= 42.24 \times 10^5 \text{ Pa} \end{aligned}$$

5-2. 解: 1) 一般计算

$$Q = \frac{90 \times 1000}{900 \times 3600} = 0.02778 \text{ m}^3/\text{s}$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.02778}{\frac{\pi}{4} \times 0.2^2} = 0.884 \text{ m/s}$$

$$Re_1 = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{0.884 \times 0.2}{1.09 \times 10^{-4}} = 1622$$

$$Re_2 = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{0.884 \times 0.2}{0.42 \times 10^{-4}} = 4209.52$$

2) 求沿程能量损失

当  $Re_1 < 2000$ , 层流。

$$\lambda_1 = \frac{0.3164}{Re_1} = \frac{64}{1622} = 0.03946$$

$$h_{f1} = \lambda_1 \frac{L V^2}{D 2g} = 0.03946 \times \frac{3000 \times 0.884^2}{0.2 \times 2 \times 9.8} = 23.6 \text{ m}$$

当  $4000 < Re_2 < 26.98 \left( \frac{D}{\Delta} \right)^{\frac{8}{3}} = 26.98 \left( \frac{0.2}{0.2} \right)^{\frac{8}{3}} = 7.238 \times 10^5$ , 在水力光滑区。故

$$\lambda_2 = \frac{0.3164}{(Re_2)^{\frac{1}{4}}} = \frac{0.3164}{(4209.52)^{\frac{1}{4}}} = 0.0393$$

$$h_{f2} = \lambda_2 \frac{L V^2}{D 2g} = 0.0393 \times \frac{3000 \times 0.884^2}{0.2 \times 2 \times 9.8} = 23.5 \text{ m}$$

5-3. 解: 1) 求流量

$$i = \frac{h_f}{L} \Rightarrow h_f = iL = 0.005 \times 50000 = 250 \text{ m}$$

取流量  $Q = 215 \text{ m}^3/h$

$$Q = \frac{215}{3600} = 0.05972 \text{ m}^3/s$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.05972}{\frac{\pi}{4} \times 0.257^2} = 1.1513 \text{ m/s}$$

$$\lambda = \frac{h_f}{L V^2} = \frac{250}{50000 \times \frac{1.1513^2}{2 \times 9.8}} = 0.019$$

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{1.1513 \times 0.257}{1.2 \times 10^{-6}} = 2.4656 \times 10^5$$

$$26.98 \left( \frac{257}{0.15} \right)^{\frac{5}{7}} = 1.339 \times 10^5 < \text{Re} < 4160 \left( \frac{257}{2 \times 0.15} \right)^{0.85} = 12.94 \times 10^5$$

故采用下式计算沿程阻力系数：

$$\frac{1}{\sqrt{\lambda'}} = 1.74 - 0.87 \ln \left( \frac{2 \times 0.15}{257} + \frac{18.7}{2.4656 \times 10^5 \times \sqrt{\lambda'}} \right)$$

$$\text{解得 } \lambda' = 0.019, \quad \lambda = \lambda', \quad \text{流量满足要求。} \quad Q_m = \frac{24 \rho Q}{1000} = \frac{24 \times 800 \times 215}{1000} = 4128 \text{ l/d.}$$

2) 求泵压

根据伯努里方程，有：

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

根据题意， $V_1 = V_2 = V$ ，故泵压为：

$$\begin{aligned} p_1 &= p_2 + \rho g h_f \\ &= 1.5 \times 10^5 + 800 \times 9.8 \times 250 \\ &= 21.1 \times 10^5 \text{ Pa} \end{aligned}$$

5-4. 解：取管径  $D = 157 \text{ mm}$

$$Q = \frac{40 \times 1000}{900 \times 3600} = 0.01234 \text{ m}^3/\text{s}$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.01234}{\frac{\pi}{4} \times 0.157^2} = 0.6377 \text{ m/s}$$

$$\lambda = \frac{2iDg}{V^2} = \frac{2 \times 0.0095 \times 0.157 \times 9.8}{0.6377^2} = 0.0719$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{0.6377 \times 0.157}{1.125 \times 10^{-4}} = 889.96$$

当  $\text{Re} < 2000$ ，层流。

$$\lambda' = \frac{64}{\text{Re}} = \frac{64}{889.96} = 0.0719$$

$\lambda = \lambda'$ ，管径满足要求。

5-5. 解：根据伯努里方程，有：

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

根据题意不计高差， $V_1 = V_2 = V$ ，故：

$$h_f = \frac{p_1 - p_2}{\rho g}$$

$$h_{f1} = \frac{p_1 - p_2}{\rho_1 g} = \frac{50 \times 10^5}{893 \times 9.8} = 571.34 \text{ m}$$

$$h_{f2} = \frac{p_1 - p_2}{\rho_2 g} = \frac{50 \times 10^5}{900 \times 9.8} = 566.89 \text{ m}$$

$$h_{f1} = \frac{0.3164}{\left(\frac{\rho_1 V_1 D}{\mu_1}\right)^{0.25}} \frac{L V_1^2}{D 2g}$$

$$h_{f2} = \frac{0.3164}{\left(\frac{\rho_2 V_2 D}{\mu_2}\right)^{0.25}} \frac{L V_2^2}{D 2g}$$

$$\frac{h_{f1}}{h_{f2}} = \frac{\frac{0.3164}{\left(\frac{\rho_1 V_1 D}{\mu_1}\right)^{0.25}} \frac{L V_1^2}{D 2g}}{\frac{0.3164}{\left(\frac{\rho_2 V_2 D}{\mu_2}\right)^{0.25}} \frac{L V_2^2}{D 2g}} = \left(\frac{\frac{\rho_2}{\rho_1}}{\frac{V_2}{V_1}}\right)^{0.25} = \left(\frac{\rho_2 \mu_1}{\rho_1 \mu_2}\right)^{0.25} \left(\frac{V_1}{V_2}\right)^{0.75}$$

$$\frac{Q_1}{Q_2} = \frac{V_1}{V_2} = \left( \frac{\frac{\rho_2}{\rho_1} \left(\frac{V_1}{V_2}\right)^{0.25}}{\left(\frac{\rho_2 \mu_1}{\rho_1 \mu_2}\right)^{0.25}} \right)^{\frac{1}{0.75}} = \left( \frac{\frac{571.34}{566.89}}{\left(\frac{900 \times 0.2}{893 \times 0.4}\right)^{0.25}} \right)^{\frac{1}{0.75}} = 1.1078$$

$$\frac{Q_1 - Q_2}{Q_1} = \frac{1.1078 Q_2 - Q_2}{1.1078 Q_2} = \frac{1.1078 - 1}{1.1078} = 0.0973 = 9.73\% \text{ (降低)}$$

5-6. 解: 1) 根据串联管路, 计算沿程损失

$$V_1 = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{25 \times 10^{-3}}{\frac{\pi \times 0.25^2}{4}} = 0.5093 \text{ m/s}$$

$$V_2 = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{25 \times 10^{-3}}{\frac{\pi \times 0.2^2}{4}} = 0.7958 \text{ m/s}$$

$$\begin{aligned}
 h_f &= \lambda_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + \lambda_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} \\
 &= 0.025 \times \frac{1000}{0.25} \times \frac{0.5093^2}{2 \times 9.8} + 0.026 \times \frac{500}{0.2} \times \frac{0.7958^2}{2 \times 9.8} \\
 &= 3.424 \text{m}
 \end{aligned}$$

2) 根据伯努里方程, 计算 H

$$z_0 + \frac{p_0}{\rho g} + \frac{V_0^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

根据题意,  $p_0 = p_2$ ,  $V_0 = 0$ 。故:

$$H = z_0 - z_2 = \frac{V_2^2}{2g} + h_f = \frac{0.7958^2}{2 \times 9.8} + 3.424 = 3.456 \text{m}$$

5-7. 解: 1) 计算各管中流量

$$V_1 = \frac{Q_1}{\frac{\pi}{4} D_1^2}, \quad V_2 = \frac{Q_2}{\frac{\pi}{4} D_2^2}, \quad V_3 = \frac{Q_3}{\frac{\pi}{4} D_3^2}$$

$$h_{f1} = \lambda_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = \lambda_1 \frac{L_1}{D_1} \frac{Q_1^2}{2g \left( \frac{\pi}{4} D_1^2 \right)} = \frac{8}{\pi^2 g} \lambda_1 \frac{L_1}{D_1^5} Q_1^2$$

$$h_{f23} = \frac{8}{\pi^2 g} \left( \lambda_2 \frac{L_2}{D_2^5} + \lambda_3 \frac{L_3}{D_3^5} \right) Q_2^2$$

根据并联管路的特点, 建立方程:

$$h_{f23} = h_{f1}$$

$$\frac{8}{\pi^2 g} \left( \lambda_2 \frac{L_2}{D_2^5} + \lambda_3 \frac{L_3}{D_3^5} \right) Q_2^2 = \frac{8}{\pi^2 g} \lambda_1 \frac{L_1}{D_1^5} Q_1^2$$

$$Q_1 = Q_2 \sqrt{\frac{\lambda_2 \frac{L_2}{D_2^5} + \lambda_3 \frac{L_3}{D_3^5}}{\lambda_1 \frac{L_1}{D_1^5}}} = Q_2 \sqrt{\frac{0.024 \times \frac{900}{0.3^5} + 0.025 \times \frac{300}{0.25^5}}{0.025 \times \frac{1000}{0.25^5}}} = 0.8045 Q_2$$

$$Q = Q_1 + Q_2 \Rightarrow Q_2 = \frac{Q}{1.8045} = \frac{100}{1.8045} = 55.417 \text{ (L/s)}$$

$$Q_1 = Q - Q_2 = 100 - 55.417 = 44.583 \text{ (L/s)}$$

2) 计算 AB 间水头损失

$$V_1 = \frac{Q_1}{\frac{\pi}{4} D_1^2} = \frac{44.583 \times 10^{-3}}{\frac{\pi}{4} \times 0.25^2} = 0.908 \text{ m/s}$$

$$h_{f1} = h_{f1} = \lambda_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = 0.025 \times \frac{1000}{0.25} \times \frac{0.908^2}{2 \times 9.8} = 4.21 \text{ m}$$

5-8. 解: 1) 计算各管中流量

$$V_0 = \frac{Q_0}{\frac{\pi}{4} D_0^2} = \frac{100 \times 10^{-3}}{\frac{\pi}{4} \times 0.25^2} = 2.037 \text{ m/s}$$

$$h_{f0} = \lambda_0 \frac{L_0}{D_0} \frac{V_0^2}{2g} = 0.025 \times \frac{500}{0.25} \times \frac{2.037^2}{2 \times 9.8} = 10.587 \text{ m}$$

根据分支管路的特点, 在接点 C 处的总能头对各支管均相等。因此

$$z_A + \frac{p_A}{\rho g} + \frac{V_A^2}{2g} - h_{f1} = z_0 + \frac{p_0}{\rho g} + \frac{V_0^2}{2g} + h_{f0}$$

$$h_{f1} = z_A - z_0 - \frac{V_0^2}{2g} - h_{f0} = 20 - 5 - \frac{2.037^2}{2 \times 9.8} - 10.587 = 4.201 \text{ m}$$

$$z_B + \frac{p_B}{\rho g} + \frac{V_B^2}{2g} - h_{f2} = z_0 + \frac{p_0}{\rho g} + \frac{V_0^2}{2g} + h_{f0}$$

$$h_{f2} = z_B - z_0 - \frac{V_0^2}{2g} - h_{f0} = 19 - 5 - \frac{2.037^2}{2 \times 9.8} - 10.587 = 3.201 \text{ m}$$

$$h_{f1} = \lambda_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} \Rightarrow V_1 = \sqrt{\frac{h_{f1}}{\lambda_1 \frac{L_1}{D_1} \frac{1}{2g}}} = \sqrt{\frac{4.201}{0.029 \times \frac{500}{0.2} \times \frac{1}{2 \times 9.8}}} = 1.1357 \text{ m/s}$$

$$Q_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} \times 0.2^2 \times 1.1357 = 0.03568 \text{ m}^3/\text{s} = 35.68 \text{ L/s}$$

$$Q_2 = Q_0 - Q_1 = 100 - 35.68 = 64.32 \text{ L/s}$$

2) 计算管径  $D_2$

$$h_{f2} = \frac{8}{\pi^2 g} \lambda_2 \frac{L_2}{D_2^2} Q_2^2$$

$$D_2 = \sqrt{\frac{\frac{8}{\pi^2 g} \lambda_2 L_2 Q_2^2}{h_{f2}}} = \sqrt{\frac{\frac{8}{\pi^2 \times 9.8} 0.026 \times 300 \times 0.06432^2}{3.201}} = 0.242 \text{ m}$$

5-9. 解: 1) 求第一种情况的流量

根据伯努里方程, 有

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$h_f = z_1 - z_2 + \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 30 + \frac{(50-2) \times 10^5}{860 \times 9.8} = 599.53 \text{ m}$$

$$\text{水力坡度 } i = \frac{h_f}{L} = \frac{599.53}{50000} = 0.012$$

取流量  $Q = 332.63 \text{ m}^3 / \text{h}$

$$Q = \frac{330}{3600} = 0.0924 \text{ m}^3 / \text{s}$$

$$V = \frac{Q}{\frac{\pi}{4} D^2} = \frac{0.0924}{\frac{\pi}{4} \times 0.257^2} = 1.781 \text{ m/s}$$

$$\lambda = \frac{h_f}{L V^2} = \frac{599.53}{\frac{50000}{D 2g} \times \frac{1.781^2}{0.257^2 \times 9.8}} = 0.019$$

$$Re = \frac{\rho V D}{\mu} = \frac{1.781 \times 0.257}{6 \times 10^{-4}} = 0.7629 \times 10^5$$

当  $Re < 10^5$ , 在水力光滑区。故

$$\lambda' = \frac{0.3164}{(Re)^{1/4}} = \frac{0.3164}{(0.7629 \times 10^5)^{1/4}} = 0.019$$

$\lambda = \lambda'$ , 流量满足要求。

2) 求第二种情况的流量

$$\begin{aligned} h_f &= \lambda_1 \frac{L_1 V_1^2}{D_1 2g} + \lambda_2 \frac{L_2 V_2^2}{D_2 2g} \\ &= \lambda_1 \frac{L_1 V_1^2}{D_1 2g} + \lambda_2 \frac{L_2 V_2^2}{D_2 2g} \\ &= \frac{0.3164}{\left(\frac{V_1 D_1}{V}\right)^{0.25}} \frac{L_1 V_1^2}{D_1 2g} + \frac{0.3164}{\left(\frac{V_2 D_2}{V}\right)^{0.25}} \frac{L_2 V_2^2}{D_2 2g} \\ &= \frac{0.3164 V^{0.25}}{2g} \left( \frac{L_1}{D_1^{1.25}} + \frac{L_2}{D_2^{1.25}} \left( \frac{D_1}{D_2} \right)^{3.5} \right) V_1^{1.75} \end{aligned}$$

$$V_1 = \left[ \frac{h_f}{\frac{0.3164V^{0.25}}{2g} \left( \frac{L_1}{D_1^{1.25}} + \frac{L_2}{D_2^{1.25}} \left( \frac{D_1}{D_2} \right)^{3.5} \right)} \right]^{\frac{1}{1.75}}$$

$$V_1 = \left[ \frac{599.53}{\frac{0.3164 \times (6 \times 10^{-6})^{0.25}}{2 \times 9.8} \left( \frac{40000}{0.257^{1.25}} + \frac{10000}{0.305^{0.125}} \left( \frac{0.257}{0.305} \right)^{3.5} \right)} \right]^{\frac{1}{1.75}} = 1.90545 \text{ m/s}$$

$$Q = 3600 \frac{\pi}{4} D^2 V_1 = 3600 \times \frac{\pi}{4} \times 0.257^2 \times 1.90545 = 355.84 \text{ m}^3/\text{h}$$

5-10. 解：1) 求管路并联的沿程损失

根据题 5-9 的结果，在相同管径相同长度管线并联时，支管流量为  $166.315 \text{ m}^3/\text{h}$

$$Q_1 = \frac{166.315}{3600} = 0.0462 \text{ m}^3/\text{s}$$

$$V_1 = \frac{Q_1}{\frac{\pi}{4} D_1^2} = \frac{0.0462}{\frac{\pi}{4} \times 0.257^2} = 0.8905 \text{ m/s}$$

$$Re_1 = \frac{\rho V_1 D_1}{\mu} = \frac{V_1 D_1}{\nu} = \frac{0.8905 \times 0.257}{6 \times 10^{-6}} = 0.38145 \times 10^5$$

当  $Re_1 < 10^5$ ，在水力光滑区，故

$$\lambda_1 = \frac{0.3164}{(Re_1)^{\frac{1}{4}}} = \frac{0.3164}{(0.38145 \times 10^5)^{\frac{1}{4}}} = 0.02266$$

$$h_{f1} = \lambda_1 \frac{L_1 V_1^2}{D_1 2g} = 0.226 \times \frac{10000}{0.257} \times \frac{0.8905^2}{2 \times 9.8} = 35.64 \text{ m}$$

2) 求主管路长度

根据题 5-9 的结果和计算得到的并联沿程损失，得：

$$h_f - h_{f1} = \lambda \frac{L V^2}{D 2g}$$

$$L = \frac{h_f - h_{f1}}{\lambda \frac{V^2}{D 2g}} = \frac{599.53 - 35.64}{\frac{0.019}{0.257} \times \frac{1.781^2}{2 \times 9.8}} = 47130.5 \text{ m}$$

可延长 4730.5m。

5-12. 解：1) 求支管 1 的水头损失

根据题意，支管 1 的流量为  $50m^3/h$

$$Q_1 = \frac{50}{3600} = 0.01389m^3/s$$

$$V_1 = \frac{Q_1}{\frac{\pi}{4} D_1^2} = \frac{0.01389}{\frac{\pi}{4} \times 0.1^2} = 1.7685m/s$$

$$Re_1 = \frac{\rho V_1 D_1}{\mu} = \frac{V_1 D_1}{\nu} = \frac{1.7685 \times 0.1}{1 \times 10^{-6}} = 1.7685 \times 10^5$$

$$26.98 \left( \frac{100}{0.15} \right)^{\frac{5}{7}} = 0.4538 \times 10^5 < Re < 4160 \left( \frac{100}{2 \times 0.15} \right)^{0.85} = 5.8015 \times 10^5$$

$Re_1$  均在上述范围，故采用下式计算沿程阻力系数：

$$\frac{1}{\sqrt{\lambda_1}} = 1.74 - 0.87 \ln \left( \frac{2 \times 0.15 + \frac{18.7}{1.7685 \times 10^5 \times \sqrt{\lambda_1}}}{100} \right)$$

解得  $\lambda_1 = 0.02289$ 。支管 1 的水头损失：

$$h_{w1} = \left( 0.02289 \times \frac{0.8}{0.1} + 4 \times 0.5 + 0.4 \right) \times \frac{1.7685^2}{2 \times 9.8} = 1.04m$$

2) 求干管 2 的水头损失

$$Q_2 = \frac{2 \times 50}{3600} = 0.02778m^3/s$$

$$V_2 = \frac{Q_2}{\frac{\pi}{4} D_2^2} = \frac{0.02778}{\frac{\pi}{4} \times 0.156^2} = 1.453m/s$$

$$Re_2 = \frac{\rho V_2 D_2}{\mu} = \frac{V_2 D_2}{\nu} = \frac{1.453 \times 0.156}{1 \times 10^{-6}} = 2.267 \times 10^5$$

$$26.98 \left( \frac{100}{0.15} \right)^{\frac{5}{7}} = 0.4538 \times 10^5 < Re < 4160 \left( \frac{100}{2 \times 0.15} \right)^{0.85} = 5.8015 \times 10^5$$

$Re_2$  均在上述范围，故采用下式计算沿程阻力系数：

$$\frac{1}{\sqrt{\lambda}} = 1.74 - 0.87 \ln \left( \frac{2 \times 0.15 + \frac{18.7}{2.267 \times 10^5 \times \sqrt{\lambda}}}{156} \right)$$

解得  $\lambda = 0.02065$ 。干管 2 的水头损失

$$h_{n2} = \left( 0.02065 \times \frac{100}{0.156} + 3 + 0.4 + 1.7 \right) \times \frac{1.543^2}{2 \times 9.8} = 2.227 \text{ m}$$

3) 求油管总水头损失

$$h_u = h_{u4} + h_{n2} = 1.04 + 2.227 = 3.267 \text{ m}$$

5-13. 解: 1) 求各分支管路的流速

根据分支管路的特点, 在接点 B 处的总能头对各支管均相等。因此

$$z_E + \frac{p_E}{\rho g} + \frac{V_E^2}{2g} + h_{jBE} = z_D + \frac{p_D}{\rho g} + \frac{V_D^2}{2g} + h_{jBD}$$

$$\frac{V_E^2}{2g} + h_{jBE} = \frac{V_D^2}{2g} + h_{jBD}$$

在不考虑速度水头时, 有:

$$h_{jBE} = h_{jBD}$$

$$V_E = \left[ \frac{\left( \frac{L_2}{d_1^{1.25}} \left( \frac{d_2}{d_1} \right)^{3.5} \frac{V_D}{d_1^{1.25}} \right)^{1/75}}{\frac{V_D}{d_1^{1.25}}} \right]^{1/75} V_D$$

$$= \left[ \frac{\left( \frac{10}{0.203^{1.25}} \left( \frac{100}{203} \right)^{3.5} + \frac{5}{0.1^{1.25}} \right)^{1/75}}{\frac{5}{0.1^{1.25}}} \right] V_D$$

$$= 1.039 V_D$$

$$Q = \frac{\pi}{4} d_2^2 (V_E + V_D) = \frac{\pi}{4} d_2^2 (1.039 V_D + V_D) = 2.039 \frac{\pi}{4} d_2^2 V_D$$

$$V_D = \frac{Q}{2.039 \frac{\pi}{4} d_2^2} = \frac{\frac{2 \times 50}{3600}}{2.039 \times \frac{\pi}{4} \times 0.1^2} = 1.734 \text{ m/s}$$

$$V_E = 1.039 V_D = 1.039 \times 1.734 = 1.802 \text{ m/s}$$

2) 求分支管路的沿程损失

$$Re_E = \frac{\rho V_E d_2}{\mu} = \frac{V_E d_2}{V} = \frac{1.802 \times 0.1}{4 \times 10^{-6}} = 4.505 \times 10^4$$

$$\lambda_{BE} = \frac{0.3164}{(\text{Re}_E)^{0.25}} = \frac{0.3164}{(4.505 \times 10^4)^{0.25}} = 0.0217$$

$$h_{jBE} = \lambda_{BE} \frac{l_1}{d_2} \frac{V_E^2}{2g} = 0.0217 \times \frac{5}{0.1} \times \frac{1.802^2}{2 \times 9.8} = 0.18m$$

3) 求干管路的沿程损失

$$V_A = \frac{Q}{\frac{\pi}{4} d_1^2} = \frac{\frac{2 \times 50}{3600}}{\frac{\pi}{4} \times 0.203^2} = 0.858m/s$$

$$\text{Re}_A = \frac{\rho V_A d_1}{\mu} = \frac{V_A d_1}{\nu} = \frac{0.858 \times 0.203}{4 \times 10^{-6}} = 4.355 \times 10^4$$

$$\lambda_{AB} = \frac{0.3164}{(\text{Re}_A)^{0.25}} = \frac{0.3164}{(4.355 \times 10^4)^{0.25}} = 0.0219$$

$$h_{jAB} = \lambda_{AB} \frac{L_1}{d_1} \frac{V_A^2}{2g} = 0.0219 \times \frac{250}{0.203} \times \frac{0.858^2}{2 \times 9.8} = 1.013m$$

4) 求泵的出口压强

$$\begin{aligned} p_A &= \rho g [1.05 \times (h_{jAB} + h_{jBE}) + 4] \\ &= 800 \times 9.8 \times 1.05 \times (1.013 + 0.18) + 4 \\ &= 0.412 \times 10^6 Pa \end{aligned}$$